### Chapter 8.4  Check Your Understanding

#### Exercises 1–10  True or False. Give reasons.

1. When $n$ is 1, 2, 3, 4, or 5, the sum of the first $n$ odd positive integers is equal to $n^2$.

   **Answer:**

   True; $1 = 1^2$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, $1 + 3 + 5 + 7 = 4^2$, $1 + 3 + 5 + 7 + 9 = 5^2$.

2. When $n$ is 1, 2, 3, 4, or 5, the sum of the first $n$ even positive integers is equal to $n(n + 1)$.

   **Answer:**

   True; $2 = 1(1 + 1)$, $2 + 4 = 2(2 + 1)$, $2 + 4 + 6 = 3(3 + 1)$, $2 + 4 + 6 + 8 = 4(4 + 1)$, $2 + 4 + 6 + 8 + 10 = 5(5 + 1)$.

3. If $f(n) = n^2 - n + 17$, then $f(n)$ is a prime number for $n = 1, 2, 4, 8$, and 17.

   **Answer:**

   False; for $n = 17$, $n^2 - n + 17 = 17^2 - 17 + 17 = 17^2$.

4. If $f(n) = n^2 + n$, then $f(n)$ is an even number for every positive integer $n$.

   **Answer:**

   True; $f(n) = n^2 + n = n(n + 1)$. Since $n$ and $n + 1$ are consecutive integers, then one must be even and the other odd, and so the product must be even.

5. Evaluating the expressions $(n + 1)^2$ and $2^n$ for $n = 1, 2, 3, 4, 5$, and 6, it is reasonable to conclude that $(n + 1)^2 > 2^n$ for every positive integer.

   **Answer:**

   False; for $n = 6$, $(6 + 1)^2 > 2^6$, or $49 > 64$ is false.

6. For every positive integer $n$, $3^n + 1$ is an even number.

   **Answer:**

   True; $3^n$ is odd for every $n$ and so $3^n + 1$ must be even.
7. For every positive integer $n$, the units digit of $5^n - 1$ is 4.
   **Answer:**
   True; the units digit for $5^n$ is 5 for every $n$, so the units digit for $5^n - 1$ must be 4.

8. When $n$ is 1, 2, 3, or 4, $5^n + 1$ is not divisible by 4.
   **Answer:**
   True; evaluate $5^n + 1$ for $n = 1, 2, 3,$ and 4. None of the resulting numbers is divisible by 4.

9. For every positive integer $n$, the units digit of $2^n$ is 2, 4, or 8.
   **Answer:**
   False; when $n$ is 4, $2^4 = 2^4 = 16$.

10. For every positive integer $n$, the units digit of $4^n - 1$ is 3 or 5.
    **Answer:**
    True; the units digit of $4^n$ is either 4 or 6 for every positive integer $n$. Therefore, the units digit of $4^n - 1$ must be 3 or 5.