# Chapter 3.2  Check Your Understanding

**Draw a graph whenever helpful.**

<table>
<thead>
<tr>
<th>Exercises 1–6  True or False. Give reasons.</th>
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<tr>
<td>1. The function ( p(x) = 4x^3 - x ) has three real zeros.  ( \text{Answer:} ) True; ( p(x) = 4x^3 - x = x(4x^2 - 1) = x(2x + 1) ). The zeros are 0, ± ( \frac{1}{2} ).</td>
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<td>2. The positive zero of ( f(x) = x^3 - 3x ) is less than 1.73.  ( \text{Answer:} ) False; ( x^3 - 3x = 0, x(x^2 - 3) = 0 ). The zeros are 0, ± ( \sqrt{3} ). The positive zero ( \sqrt{3} (\approx 1.732) ) is greater than 1.73.</td>
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<td>3. For ( f(x) = x^3 - 1.6x^2 - 8.52x + 15.84 ), since ( f(2) ) and ( f(3) ) are positive, then ( f ) contains no zeros between 2 and 3.  ( \text{Answer:} ) False; there could be a smaller interval where there is a sign change. Try ( x = 2.2 ).</td>
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<td>4. The equation ( 2x^3 - 5x^2 + 4x - 1 = 0 ) has no rational roots.  ( \text{Answer:} ) False; ( f(1) = 2 - 5 + 4 - 1 = 0 ), so 1 is a rational zero.</td>
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<td>5. The function ( f(x) = (3x - 2)(x^2 - 2x - 4) ) has exactly one real zero.  ( \text{Answer:} ) False; ( f(x) = (3x - 2)(x^2 - 2x - 4) ) so ( x = \frac{2}{3} ) or ( x = \frac{2\pm\sqrt{4-4(-4)}}{2} = 1 \pm \sqrt{5} ).</td>
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<td>6. When ( x^3 - 2x^2 + 3x - 16 ) is divided by ( x - 3 ), then the remainder is 2.  ( \text{Answer:} ) True; carry out the division or evaluate ( x^3 - 2x^2 + 3x - 16 ) at ( x = 3 ). In either case ( r = 2 ).</td>
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| Exercises 7–10  Fill in the blank so that the resulting statement is true. |
7. If \( x^3 + 2x^2 + 1 = (x + 1)(x^2 + x - 1) + r \) for every value of \( x \), then \( r = \) ______.

**Answer:**
Replace \( x \) by \(-1\) in the given equation and see that \( r = 2 \).

8. The number of rational zeros of
\[
f(x) = (x^2 - 2)(x^2 - 2x + 3)
\]
is ______.

**Answer:**
\( x^2 - 2 = 0 \) gives \( x = \pm \sqrt{2} \) (not rational), \( x^2 - 2x + 3 = 0 \) gives \( x = 1 \pm \sqrt{2}i \). There are no rational roots.

9. The number of real roots of
\[
(x^2 - 2)(x^2 - 2x + 3) = 0
\]
is ______.

**Answer:**
\( f(x) = (x^2 - 2)(x^2 - 2x + 3) \) so \( x = \pm \frac{\sqrt{4(2)}}{2} = \pm \sqrt{2} \) or \( x = 2 \pm \frac{\sqrt{4 - 12}}{2} = 1 \pm \sqrt{2}i \). Therefore \( f \) has two real roots, \( \pm \sqrt{2} \).

10. If \( x^3 - 2x^2 + 3x - 5 \) is divided by \( x + 1 \), then the remainder is ______.

**Answer:**
Substitute \(-1\) for \( x \) in \( x^3 - 2x^2 + 3x - 5 \). \((-1)^3 - 2(-1)^2 + 3(-1) - 5 = -5 \). The remainder is \(-5\).