In Chapter 1 we consider the nature of mathematics, where mathematics comes from, and how it is used. This chapter lays a foundation for the entire book. Section 1.1 describes how mathematical models represent real-world problems, including calculator use and approximations. Sections 1.2 and 1.3 review terminology and the properties of numbers related to ordering and absolute values. Section 1.4 introduces the ideas of graphs and their uses, both on a number line and in the plane. Section 1.5 reviews some of the techniques from elementary algebra, how these techniques relate to graphing technology, and how they allow us to find solution sets for a variety of kinds of open sentences. The final section demonstrates how to approach and formulate a number of different problems, introducing techniques that are useful throughout the rest of the book and all of the study of mathematics.

1.1 Mathematics Models the World

Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them.

Joseph Fourier

What Is Mathematics?

Consider these situations and note what they have in common:

1. At the edge of the Beaufort Sea, north of the Arctic Circle, a dozen adults of the Inuit people are tossing a young boy aloft on a human-powered trampoline made of a blanket.
2. From the observation deck of the Sears Tower (the world’s tallest building at 1454 feet) a visitor can see nearly six miles further out into Lake Michigan than someone at the top of the John Hancock Center (1127 feet tall).

3. A pilot of a Goodyear blimp heading south over Lake Okeechobee at 5300 feet wants to estimate the time remaining before visual contact with the Orange Bowl, where a football game is to be televised.

Each of these situations deals with the curvature of the earth’s surface and the fact that it is possible to see farther from a higher elevation. The Inuits want to get an observer high enough to see whether the pack ice is breaking up in the spring; the Sears Tower is 327 feet taller than the Hancock Center; at an elevation just over a mile, how far can the blimp pilot see?

Mathematics strips away the differences in these situations and finds one simple model to describe common key features. Figure 1 shows a cross section of the earth as a circle with center at $O$ and radius $r$. Our model assumes a spherical earth, a fairly good approximation of the truth. From point $A$ located $x$ feet above the earth, the line of sight extends to $B$. (Line $AB$ is tangent to the circle and hence perpendicular to radius $OB$.)

All of the situations listed above fit in this structure. The Inuit boy, the visitor to the top of a skyscraper, and the blimp pilot could each be seen as located at point $A$ for different values of $x$. For any given $x$, applying the Pythagorean theorem (discussed in Section 1.4) to the right triangle $AOB$ gives the corresponding distance $s$ to the horizon.

\[
\begin{align*}
    r^2 + s^2 &= (r + x)^2, \text{ where } x, r, \text{ and } s \text{ are in miles.} \\
    r^2 + s^2 &= r^2 + 2rx + x^2 \\
    s^2 &= 2rx + x^2 \\
    s &= \sqrt{2rx + x^2}.
\end{align*}
\]

Part of the power of mathematics comes from its capacity to express in a single sentence truths about several seemingly diverse situations. The solution to one equation automatically applies to any other application that gives rise to the same equation. The expression for $s$ can be used to solve any of the problems listed above. See Exercises 39–44.

Mathematics and the Real World

Much of the importance and vitality of mathematics comes from its relationship with the world around us. Humans invented numbers to count our sheep; we created rules for addition and multiplication as we needed to barter or compare land holdings. As human understanding of the world grew more sophisticated, mathematical tools grew as well. Sometimes mathematical curiosity led people in unexpected directions and their explorations became important for their own sake.

Mathematics is a lively part of our intellectual heritage. Some of the most intriguing and challenging mathematical investigations grew out of attempts to answer seemingly innocuous questions or understand simple observations. The most lasting and significant human achievements are direct consequences of our desire to understand and control the world.
1.1 Mathematics Models the World

Mathematics and Mathematical Models

When we encounter a problem whose solution involves the use of mathematics, we must decide how much detail is essential. In the line-of-sight examples the solution assumed the earth as a perfect sphere. The differences between that mathematician’s earth and the actual globe are substantial. The equation for the distance to the horizon (s miles) implies that someone could see more than 40 miles from the top of either the Sears Tower or the Hancock Center; on a clear day someone in Chicago might want to check that conclusion.

In a mountain valley ringed by peaks that rise several thousand feet, it isn’t possible to see 40 miles in any direction, even from 1500 feet up. Does that invalidate our mathematical model? Of course not; we must know something about particulars when we interpret a result. Questions about the way the world works frequently require simplifying assumptions to make the problems more tractable. See the Historical Note, Mathematical Models and Gravity (p. 135).

Technology

We assume that every student has access to some kind of graphing technology that permits graphing functions. Yours may be as simple as a graphing calculator or as complex as sophisticated computer software. We use the language of graphing calculators in this text, but you can use any available technology to do the work. If you are working with technology that is new to you, perhaps the most important thing is to experiment freely so that you become comfortable and confident with your own tools. Verify every computation in our examples. Talk with others about what works and what doesn’t. Make sure that you can produce the same kinds of pictures that we show in the text.

Each graphing calculator and computer graphing software package is different; display screens have different proportions, and commands and syntax vary. We cannot give instructions to fit every kind of machine, but it should be possible to duplicate our computations and calculator graphs on almost any kind of graphing technology you have available. In our Technology Tips we make suggestions that may be helpful. If it seems that your calculator won’t do something we are describing, discuss it with your instructor, look at your owner’s manual, and ask classmates. There may be another way to get around the problem.

Calculators and computers have become incredibly powerful, but they remain limited. While they can do wonders, they may still properly be called “Smart-Stupids,” a name coined by Douglas Hofstadter. However amazing their computing power, the machine is not smart enough to know that we meant to press $\frac{1}{4}$ when we pressed the $\frac{1}{3}$ key.

Approximate Numbers and Significant Digits

When we use mathematics to model the real world, we have to realize that measurements of physical quantities can be only approximations. A biologist may be able to count exactly the number of eggs in a bird’s nest, but comparing the volume or weight of two eggs requires approximate numbers, since any number we use is only as good as our measuring device. We also use approximate numbers when we need a decimal form for a number such as $\sqrt{3}$.

Questions involving approximations entail decisions about the tolerable degree of error. Error tolerance decisions usually hinge on concerns other than mathematics, but all of us must make such decisions in working problems that involve measurements or when we use calculators. We need guidelines.
Chapter 1 Basic Concepts: Review and Preview

Perhaps the greatest problem in working with calculators is interpreting displayed results. When we enter data, the calculator returns so many digits so quickly and easily that we may think we have gained more information than we really have. This difficulty can be illustrated by an example from a recent calculus text. The book derives an equation for the volume of a pyramid, as shown in Figure 2, and then applies the formula to find the volume of the Great Pyramid of Cheops. The original dimensions are given (approximately) as

$s = 754$ feet and $h = 482$ feet.

When we substitute these numbers into the formula, a calculator immediately displays $91341570.67$, from which the authors conclude that the volume is “approximately $91,341,571$ cubic feet.” In the following example, we illustrate why we are not justified in rounding to the nearest cubic foot, even if the values for $s$ and $h$ are measured to the nearest foot.

**EXAMPLE 1 Appropriate rounding**

Assuming that the height $h$ and side length $s$ are measured to the nearest foot, giving 482 feet for $h$ and 754 for $s$, how much variation can this leave in the computed volume, using $V = \frac{1}{3}s^2h$?

**Strategy:** Let $V_0$ be the volume using the smaller values of $s$ and $h$, while $V_1$ is the volume using the larger values of $s$ and $h$. Compute $V_0$ and $V_1$, and then compare the results.

**Solution**

To say the linear measurements are correct to the nearest foot means that they satisfy the inequalities

$753.5 < s < 754.5 \quad \text{and} \quad 481.5 < h < 482.5$.

Using the smaller values for $s$ and $h$ gives

$V_0 = \frac{(753.5)^2(481.5)}{3} \approx 91,125,841.12$.

The upper values for $s$ and $h$ yield

$V_1 = \frac{(754.5)^2(482.5)}{3} \approx 91,557,631.87$.

The difference between $V_1$ and $V_0$ is

$V_1 - V_0 \approx 431,790.75$.

The computed and actual volumes could differ by nearly *half a million cubic feet!*

See Example 3. ▲

The world of mathematics is an *ideal* world, dealing with exact numbers and precise relationships, but mathematics also says much about the inexactitude and fuzziness of the physical world. In applying mathematics, we create a precise model to mirror an imprecise reality. Whenever mathematics delivers an answer for an applied problem, we must ask what the numbers mean and what degree of significance they have for the original problem.

**What Do the Digits Mean?**

What is the diagonal of a square that measures a mile on each side? The mathematical model of a square of side 1 has a diagonal of exactly $\sqrt{2}$. Our calculator displays 1.414213562 for $\sqrt{2}$. For a mile, what does each of these decimal places
measure? We must consider what degree of accuracy makes sense in the real world. If the sides are measured only to the nearest 10 feet, it makes no sense to pretend to have an accuracy indicated by enough digits to measure the thickness of a blade of grass! The following box shows something of the meaning of each decimal place when we talk of the decimal parts of a mile.

### Decimal parts of a mile:

<table>
<thead>
<tr>
<th>Decimal Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>Two football fields</td>
</tr>
<tr>
<td>.01</td>
<td>Width of a city street</td>
</tr>
<tr>
<td>.001</td>
<td>Height of a 5'3” person</td>
</tr>
<tr>
<td>.0001</td>
<td>6” (less than a handspan)</td>
</tr>
<tr>
<td>.00001</td>
<td>Width of a finger</td>
</tr>
<tr>
<td>.000001</td>
<td>(\frac{3}{16})” (smallest mark on a ruler)</td>
</tr>
<tr>
<td>.0000001</td>
<td>Thickness of a 1.5 sheets of paper</td>
</tr>
<tr>
<td>.00000001</td>
<td>Half the thickness of a human hair</td>
</tr>
</tbody>
</table>

### Significant Digits, Precision, and Scientific Notation

When multiplying and dividing approximate numbers, we consider **significant digits**, the digits that indicate measured accuracy. Normally zeros that serve only to locate the decimal point are not significant. These numbers each have four significant digits:

- 400.5 ft.
- 0.002596 mm.
- 1.032 km.
- 93,410,000 mi.

In scientific applications, special notation makes it easy to identify the significant digits. By moving the decimal point as needed, any positive real number can be written as a product of a number between 1 and 10 and some power of 10. A number written as such a product is said to be in **scientific notation**. We would write the four numbers above in scientific notation as follows.

- 4.005 \(\times 10^2\)
- 2.596 \(\times 10^{-3}\)
- 1.032
- 9.341 \(\times 10^7\)

Usually we do not write \(10^0\) for a number that is already between 1 and 10.

In addition and subtraction, our concern is **precision**, the question of which decimal places have meaning. If we are told that an ancient tree is 3000 years old, we consider that 3000 as a less precise number than the age of a 17 year old.

### Guidelines for computation with approximate numbers

In multiplication and division with approximate numbers, round off final results to the **least number of significant digits** in the data used. Results are no more accurate than the least accurate data; record no more significant digits than occur in any of the given data.

In addition and subtraction with approximate numbers, round off final results to the **least level of precision** in the data used.

#### EXAMPLE 2 Significant digits

Determine which digits are significant, and write the number in scientific notation.

(a) 325.6  (b) 28.40  (c) 205,000  (d) 0.00640

**Solution**

(a) All digits are significant: \(325.6 = 3.256 \times 10^2\).

(b) The last 0 does not locate the decimal point; all digits are significant: \(28.40 = 2.480 \times 10\).
(c) Without additional information, we can only assume that the first three digits are significant: 205,000 = 2.05 \times 10^5. If we had some reason to believe that 205,000 represented a measurement accurate to the nearest hundred, then four digits would be significant and we would write 205,000 = 2.050 \times 10^5.

(d) The first three zeros just locate the decimal point, but the last zero is significant: 0.00640 = 6.40 \times 10^{-3}.

**Example 3** Rounding off

Use the formula \( V = \frac{1}{3} \pi h \) to calculate the volume of the Great Pyramid of Cheops, where \( s = 754 \) feet and \( h = 482 \) feet.

**Solution**

Follow the strategy.

\[ V = \frac{(754)^2(482)}{3} \approx 91,341,570.67 = 91,300,000. \]

Hence the volume is approximately 91,300,000 cubic feet. As we would expect, the result lies well between the extreme values of \( V_0 \) and \( V_1 \) in Example 1.

**Example 4** Precision

Simplify, assuming that the numbers are approximate measurements:

(a) 2.483 + 15.4
(b) 7200 – 1720 + 32

**Solution**

(a) Since 15.4 (measured to the nearest tenth) is less precise than 2.483 (measured to the nearest thousandth), round off the sum to the precision of the less precise number.

\[ 2.483 + 15.4 = 17.883 \approx 17.9. \]

(b) The least precise of these numbers is 7200, so round off the sum to the same level of precision, to the nearest 100:

\[ 7200 – 1720 + 32 = 5512 \approx 5500. \]

**The Number \( \pi \)**

The number \( \pi \), denoted by the Greek letter \( \pi \), pops up in the most unexpected places in mathematics, several of which we encounter in this book. See the Historical Note, “The Number \( \pi \).” Most of us first meet \( \pi \) in connection with circles through its historical definition as the ratio of the circumference to the diameter of a circle. Scientific calculators have a key labeled \( \frac{\pi}{\pi} \), which approximates \( \pi \):

\[ \pi \approx 3.141592654. \]

**Example 5** Rounding off

A rectangular garden measures 23 feet by 36 feet (to the nearest foot). What is the distance \( c \) between its opposite corners? (See Figure 3.)

**Solution**

Using the Pythagorean theorem,

\[ c = \sqrt{23^2 + 36^2} \approx 42.72001873. \]

Our rule suggests stating the result to two significant digits, so the diagonal distance is 43.
1.1 Mathematics Models the World

The number π (which represents the ratio of the circumference to the diameter of a circle) has fascinated people since antiquity. The Babylonians, Chinese, and Hebrews all knew that the value of π was near 3. The Egyptians computed the area of a circle by squaring \( \frac{1}{2} \) of the diameter, implying a value for π of \( \frac{256}{81} = 3.16 \). In about 200 B.C., Archimedes used inscribed and circumscribed polygons (see figure) to get the bounds \( 3 \frac{10}{7} < \pi < 3 \frac{1}{7} \). We still use \( \frac{22}{7} \) as a convenient (and fairly good) approximation.

More than 1700 years passed before a Frenchman, Viète, significantly improved on the efforts of the Greeks. Real progress accompanied the invention of calculus. Sir Isaac Newton calculated π to 15 decimal places and confessed to a colleague, “I am ashamed to tell you how many figures I carried these calculations, having no other business at the time.” By 1706 Machin in England correctly computed π to 100 digits.

Through all this time, people were looking for a repeating pattern of digits. It wasn’t until 1761 (2000 years after the Pythagoreans proved that \( \sqrt{2} \) is not rational) that Lambert (from Germany) finally proved that π is irrational, so the pattern of digits will never repeat.

**EXAMPLE 6 Area of circle**

The radius of a circle measures 24.5 cm. What is its area?

**Solution**

The equation for the area \( A \) of a circle in terms of the radius is \( A = \pi r^2 \). Replacing \( r \) with 24.5, and rounding off to three significant digits, \( A = \pi (24.5)^2 \approx 1890 \).

The area is approximately 1890 cm\(^2\).

**EXERCISES 1.1**

**Check Your Understanding**

**Exercises 1–5 True or False. Give reasons.**

1. If \( x \) is any real number, then \( x^2 \) is positive.

2. If \( x \) is a number such that \( \frac{1}{x} < 1 \), then \( x \) must be greater than 1.

3. For all nonnegative numbers \( x \) and \( y \), \( \sqrt{x + y} \geq \sqrt{x} + \sqrt{y} \).

4. Without additional information, we must assume that the zeros in the numbers 45,000 and 0.0045 are not significant digits.

5. All of the zeros in the numbers 3.005 and 4.720 are significant digits.

**Exercises 6–10 Fill in the blank so that the resulting statement is true.**

6. In the decimal representation of the quotient \( \frac{1}{7} \), the digit in the fourth decimal place is ______.

7. The number of significant digits in
   (a) 10.2 is ______.
   (b) 1200 is ______.
   (c) 0.12 is ______.

8. If the dimensions (in inches) of a cereal box are measured to be 3.1 \( \times \) 7.2 \( \times \) 8.9, then the diagonal of the box can be calculated and the number of meaningful significant digits is ______.

9. Of the three numbers \( \pi \), \( \frac{333}{106} \), \( \frac{355}{113} \), the largest one is ______.

10. If the radius of a circle is doubled, then its area increases by a factor of ______.
Develop Mastery

Exercises 1–15 Calculator Evaluation The purpose of these exercises is to give you practice with your calculator. Many of the exercises are simple enough to solve in your head. Their real value comes from the effort to make your calculator do all the necessary steps and agree with the result in brackets. Some answers are rounded off to three decimal places.

1. (6 + 3) · 8 [72]  2. 6 · 3 + 3 · 8 [42]
3. 2 · 3² + 3 · 4² [66]  4. \( \frac{1}{2} - \frac{3}{4} \) [-0.625]
5. \( \frac{2 / 3 + 3 / 4}{7 / 8} \) [1.619]  6. \( (2 \cdot 3)^2 + (3 \cdot 4)^2 \) [180]
7. \( \frac{4.5 - 3.1}{5.6} \) [0.35]  8. \( 4.5^2 - (3.1)^2 \) [18.534]
9. \( \frac{(4.5)^2 - (3.1)^2}{5.6} \) [1.9]  10. \( \sqrt{2} + \frac{3}{2} \) [2.236]
11. \( \sqrt{2} + \frac{\sqrt{3}}{3} \) [3.146]  12. \( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \) [1.284]
13. \( \frac{1}{\sqrt{2} + \sqrt{3}} \) [0.318]  14. \( 1 + \sqrt{3^2 - 1} \) [1.276]
15. \( \sqrt{\frac{4 - \sqrt{2}}{2}} \) [1.137]

Exercises 16–17 Scientific Notation Write in scientific notation and tell which digits are significant.

16. (a) 406  (b) 40600  (c) 406.0  (d) 0.0406
17. (a) 807  (b) 8070  (c) 807.0  (d) 0.008070

Exercises 18–19 Significant Digits Tell which digits are significant, and then express in standard decimal notation.

18. (a) 3.2 \times 10^3  (b) 5.06 \times 10^{-3}
    (c) 8.400 \times 10^{-2}  (d) 3.40 \times 10^3
19. (a) 6.4 \times 10^3  (b) 7.06 \times 10^{-3}
    (c) 3.470 \times 10^{-2}  (d) 5.60 \times 10^4

Exercises 20–21 Round off to two significant digits.

20. (a) 3254  (b) 4.32  (c) 0.05642
    (d) 357894
21. (a) 80.5  (b) 0.35501  (c) 0.03618
    (d) 247631

Exercises 22–23 How Big Is a Trillion?

22. A stack of 250 dollar bills is about 1 inch high. How high would a stack of \( (a) \) 4 million  \( (b) \) 4 billion  \( (c) \) 4 trillion dollar bills reach? Make a guess before you do any calculations, such as: 250 feet, \( \frac{1}{2} \) mile, the distance from Boston to New York (230 miles), the distance from the earth to the moon (240,000 miles), the distance from the earth to the sun. (The federal debt is well over 5 trillion dollars.)
23. Would you classify as a baby, a teenager, an adult, an old person, or otherwise a person who is \( (a) \) a million seconds old?  \( (b) \) a billion seconds old?
    \( (c) \) a trillion seconds old?

Exercises 24–25 Give decimal approximations rounded off to three significant digits.

24. (a) \( \sqrt{7} \)  (b) \( \frac{2\pi}{5} \)  (c) \( \sqrt{4\pi} \)  (d) \( \sqrt{25 - \sqrt{3}} \)
25. (a) \( \sqrt{3} \)  (b) \( \frac{\pi}{5} \)  (c) \( \sqrt{25\pi} \)
    (d) \( \sqrt{17 + \sqrt{47}} \)

Exercises 26–32 Rounding Off Consider all data as measured numbers. Round off each computation to an appropriate number of significant digits.

26. \( x = 33.7, y = 2.35, z = 0.431 \). Find
    (a) \( xy \)  (b) \( yz \)  (c) \( \frac{y}{z} \).
27. Evaluate
    (a) 32.51 + 63.2  (b) 65.1 - 23.18 + 2.407
    (c) \( \sqrt{3.82^2 + 2.63^2} \).
28. Find the length of a diagonal of a rectangle with sides of 31.4 feet and 16.3 feet.
29. The radius of a circle is 3.64 feet. What is its \( (a) \) circumference?  \( (b) \) area?
30. The legs of a right triangle measure 2.4 meters and 5.8 meters. Find the \( (a) \) hypotenuse  \( (b) \) perimeter  \( (c) \) area
31. What is the volume of a sphere whose radius measures 31.4 inches? See inside cover.
32. The length of a side of a square is 2.4 yards and the radius of a circle is 4.1 feet. Which has the greater area, the square or the circle? By how much?

Exercises 33–38 Circular Motion Consider the following.

(a) Nicole rides the Sky Scraper, a gigantic Ferris wheel at Lagoon.
(b) The Galapagos islands, located near the equator, rotate with the earth.
(c) Minneapolis, located near 45° latitude, rotates with the earth.

(d) A space capsule orbits the earth.

(e) The earth orbits the sun.

Each situation can be modeled mathematically as an object traveling in a circular orbit of radius \( r \) at a fixed speed. The distance traveled in one revolution is \( 2\pi r \) (the circumference of the circle). The time \( T \) for one rotation and the rotational speed \( V \) are related by the equation \( VT = 2\pi r \).

33. The Sky Scraper carries its riders to a height of nearly 150 feet, has a wheel diameter of 137 feet, and has two speeds, 1.30 or 1.60 rotations per minute. At the slower speed, determine Nicole’s speed in
   (a) feet per second
   (b) feet per minute
   (c) miles per hour.

34. Determine Nicole’s speed, as in Exercise 33, when the Sky Scraper rotates 1.60 times per minute.

35. How fast are the giant tortoises of the Galapagos islands moving about the center of the earth (in miles per hour)? Take the radius of the earth to be 3960 miles.

36. How fast is a baseball player standing at first base in Minneapolis moving about the axis of the earth (in miles per hour)? See the diagram.

37. If the space capsule is 270 miles above the surface of the earth and makes a complete orbit in 1.70 hours, how fast is it traveling due to its rotation? The radius of the earth is 3960 miles.

38. How fast are Nicole, the Galapagos tortoises, and the entire baseball team in Minneapolis traveling (in miles per hour) about the sun? Use 93 million miles as the distance from the earth to the sun.

1.1 Mathematics Models the World

\[ x = \frac{h}{\sqrt{r^2 + h^2}} \]

the radius of the earth as \( 3.960 \times 10^3 \) miles. If \( h \) is the height in feet, then \( x = \frac{h}{\sqrt{r^2 + h^2}} \) miles.

39. How far can the Inuit boy see if he is tossed 15 feet high?

40. Compare the distances that can be seen from
   (a) the Sears Tower (\( h = 1454 \) feet)
   (b) John Hancock Center (\( h = 1127 \) feet).

41. If the air is clear, how far should the pilot of the Goodyear blimp be able to see from an elevation of 5300 feet?

42. From the top of Lagoon’s Sky Scraper ride (see Exercise 33), how far should Nicole be able to see over the Great Salt Lake?

43. Sailors follow a rule of thumb that they can see as many miles to the horizon as the square root of their height above the waterline, so a lookout in a crow’s nest 64 feet up should be able to see about 8 miles. How does this estimate compare with the figure given by the model in this section? Which do you think is more accurate? Why? See Develop Mastery Exercise 45, Section 7.1.

44. If \( s = \sqrt{2rx + x^2} \), as in this section, write an equation giving \( x \) in terms of \( s \). A lighthouse is to be built on Cape Cod on the shore of the Atlantic Ocean. How high above the ocean must the observation platform be to allow the operator to see a ship 12 miles from shore?

45. The distance between the earth and the sun is sometimes given as 93 million miles. Actually, the distance varies, from the nearest point (perihelion), about 91.4 million miles, to the furthest (aphelion), about 94.4 million miles. The speed of light is approximately 186,000 miles per second.
   (a) How long does it take light from the sun to reach the earth when the earth is at perihelion? At the aphelion? Give answers in seconds and also in minutes rounded off.
   (b) What is the difference between the times in part (a)?

Exercises 46–51 Applying Geometric Formulas Use the formulas for the following diagrams.

(a) Section cut from a sphere of radius \( R \), of depth \( d \):
\[ V = \pi d^2 (R - d/3) \]
Chapter 1  Basic Concepts: Review and Preview

46. The height $h$ and diameter $d$ of a cylindrical can of pineapple juice are measured: $h = 6\frac{1}{2}$ inches, $d = 4\frac{1}{2}$ inches. Find the volume in cubic inches and its equivalent in fluid ounces. What is the difference between your answer and 46 ounces? Explain.

47. For a soft drink cup that is supposed to hold 44 ounces, the top diameter is $4\frac{1}{8}$ and the bottom diameter is $3\frac{3}{8}$. The height of the cup is measured as $6\frac{1}{8}$. If all measurements are accurate to the nearest $\frac{1}{8}$, find the largest and smallest possible values for the volume. Is it reasonable to call the cup as a 44-ounce cup?

48. A soft drink cup is made in the shape of a frustum of a cone. If the cup is to have an upper diameter of $4\frac{1}{4}$ and the lower diameter of $3\frac{1}{2}$, what should the height be if it is to hold 32 ounces?

49. A direct mail catalog features an Oriental wok in the shape of a section of a sphere. The catalog gives dimensions that indicate $R = 6$ in., $d = 3$ in. and claims that the wok holds 2\frac{1}{2} qts. Assuming that the measurements are accurate to the nearest $\frac{1}{8}$ in., find the volume corresponding to

(a) $R = 5\frac{1}{2}$ in.  $d = 2\frac{1}{2}$ in.
(b) $R = 6\frac{1}{2}$ in.  $d = 3\frac{1}{2}$ in.

On the basis of your results in parts (a) and (b), is the catalog claim of $2\frac{1}{2}$ qts reasonable? Explain.

50. A metal barrel 18 in diameter and 30 in long is cut in half to make a trough 9 deep and 30 long.

(a) Find the volume (in cubic inches) of the resulting trough.

(b) If the diameter and length are measured accurate to the nearest quarter-inch, find the largest and smallest possible values for the volume (see Example 1).

51. Suppose the trough in Exercise 50 is cut down to make a trough of depth 4.5 in. What percent of the volume of the original is now in the shallower trough?

52. The box “Decimal Parts of a Mile” gives some familiar comparison measurements for decimal parts of a mile. Complete a similar chart for decimal parts of a kilometer.

<table>
<thead>
<tr>
<th>km</th>
<th>km</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td></td>
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<tr>
<td>0.001</td>
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</table>

1.2 REAL NUMBERS

The complexities of modern science and modern society have created a need for scientific generalists, for men (and women as well) trained in many fields of science. The habits of mind and not the subject matter are what distinguish the sciences.

Mosteller, Bode, Tukey, Winsor

Numbers occur in every phase of life. It is impossible to imagine how anyone could function in a civilized society without having some familiarity with numbers. We recognize that you have had considerable experience working with numbers, and we also assume that you know something about the language and notation of sets.