Transformations of Functions: (Part 1)

**Horizontal Shifts:**

If \( g(x) = f(x + h) \) then the graph of \( g \) can be obtained by shifting the graph of \( f \) to the left by \( h \) units. (subtract \( h \) from every \( x \)-coordinate of the graph of \( f \)).

If \( g(x) = f(x - h) \) then the graph of \( g \) can be obtained by shifting the graph of \( f \) to the right by \( h \) units. (add \( h \) from every \( x \)-coordinate of the graph of \( f \)).

Use a calculator to compare the graphs of
\[
f(x) = x^3, \quad g(x) = (x + 3)^3, \quad h(x) = (x - 3)^3
\]

Use a calculator to compare the graphs of
\[
f(x) = \frac{1}{x^2}, \quad g(x) = \frac{1}{(x + 4)^2}, \quad h(x) = \frac{1}{(x - 5)^2}
\]
**Vertical Shifts:**

If \( g(x) = f(x) + k \) then the graph of \( g \) can be obtained by shifting the graph of \( f \) up by \( k \) units. (Add \( k \) to every \( y \)-coordinate of the graph of \( f \))

If \( g(x) = f(x) - k \) then the graph of \( g \) can be obtained by shifting the graph of \( f \) down by \( k \) units. (Subtract \( k \) from every \( y \)-coordinate of the graph of \( f \))

Use a calculator to compare the graphs of

\[
f(x) = x^2, \quad g(x) = x^2 + 4, \quad h(x) = x^2 - 3
\]

Let \( f(x) = \sqrt{x} \) and let \( g(x) = \sqrt{x - 2} + 1 \)

Without graphing the functions, write a sentence that compares the graphs of \( f \) and \( g \).
Reflections:

If \( g(x) = -f(x) \) then the graph of \( g \) can be obtained by reflecting the graph of \( f \) across the \( x \)-axis. (Change the sign of every \( y \)-coordinate of the graph of \( f \))

If \( g(x) = f(-x) \) then the graph of \( g \) can be obtained by reflecting the graph of \( f \) across the \( y \)-axis. (Change the sign of every \( x \)-coordinate of the graph of \( f \))

Use a calculator to compare the graphs of \( f(x) = x^2 \), \( g(x) = x^2 + 4 \), \( h(x) = x^2 - 3 \)

Let \( f(x) = \sqrt{x} \) and let \( g(x) = \sqrt{x-2} + 1 \)

Without graphing the functions, write a sentence that compares the graphs of \( f \) and \( g \).
**Stretching and Compressing:**

Let $a$ be a positive real number.

If $g(x) = af(x)$ then the graph of $g$ can be obtained by *stretching* the graph of $f$ vertically if $a > 1$. (Multiply every $y$-coordinate of the graph of $f$ by $a$)

If $g(x) = af(x)$ then the graph of $g$ can be obtained by *compressing* the graph of $f$ vertically if $0 < a < 1$. (Multiply every $y$-coordinate of the graph of $f$ by $a$)

Use a calculator to compare the graphs of

$f(x) = \sqrt[3]{x}$, $g(x) = 2\sqrt[3]{x}$, $h(x) = \frac{1}{3}\sqrt[3]{x}$

Let $f(x) = \sqrt{x}$ and let $g(x) = -2\sqrt{x} + 2 - 1$

Without graphing the functions, write a sentence that compares the graphs of $f$ and $g$. 
Below is the graph of a function $f$. On the blank graph provided, graph the equation $y = -f(x - 2) + 3$.