Example 1 (Newton)

You drop a rock from the top of a cliff at Lake Powell that is 220 feet above the water. How fast is the rock traveling when it hits the water?
\[ s(t) = 16t^2, \quad \frac{s(t_0 + h) - s(t_0)}{h} = \]

\[ \frac{16(t_0 + h)^2 - 16t_0^2}{h} = \frac{16(t_0^2 + 2ht_0 + h^2) - 16t_0^2}{h} = \]

\[ \frac{32ht_0 + 16h^2}{h} = \frac{h(32t_0 + 16h)}{h} = \frac{\text{Av speed}}{v} \]

32t_0 + 16h when h \to 0.

What happens when h gets smaller?

Speed at time \( t_0 = 32t_0 \frac{\text{ft}}{\text{sec}} \)

Lake Powell: \( 16t^2 = 220 \)

\( t = 3.7 \text{ sec} \)

speed = \( 32(3.7) \frac{\text{ft}}{\text{sec}} = 118.4 \frac{\text{ft}}{\text{sec}} \)

\( 118.4 \frac{\text{ft}}{\text{sec}} \cdot \frac{60 \text{ sec}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mile}}{5,280 \text{ ft}} \)

\( \approx 80 \text{ mph} \)
Example 2 (Leibniz)

How can you determine the maximum and minimum values of a function? On what intervals is it increasing? On what intervals is it decreasing?

\[ y = f(x) \]
Example 3

Suppose \( y = f(x) = x^2 + 1 \).

a) Sketch the graph of \( f \).

b) Plot the points \((1, f(1))\) and \((1+h, f(1+h))\).

c) Draw the line containing these two points.

d) Find the slope of this line (secant line).

e) What happens to the slope when \( h \) is closer and closer to zero?

f) Find the equation of the line tangent to the graph of \( f \) at the point \((1, 2)\).
\[ y = f(x) = x^2 + 1 \]

\[ \text{slope of } L(h) \]
\[ = \frac{(1+h)^2 + 1 - 2}{(1+h) - 1} = \frac{1 + 2h + h^2 + 1 - 2}{h} = \frac{2h + h^2}{h} = 2 + h \text{ when } h \to 0 \]

As \( h \) gets closer and closer to 0, the secant line \( L(h) \) gets closer to the tangent line \( T \). The slope \( L(h) \) becomes a better and better approximation to the slope of \( T \) which must be 2.
\[
\frac{y - y_1}{x - x_1} = m
\]

\[
y - y_1 = m(x - x_1)
\]

**Example 3**

Point is \((1, 2)\)

Slope is 2

\[
y - 2 = 2(x - 1)
\]

or

\[
y = 2x
\]
1.1 Exercise 24

Graph the equation $|x| + |y| = 1$.

1st quadrant: $x + y = 1$ \quad x > 0, y > 0
2nd quadrant: $-x + y = 1$ \quad x < 0, y > 0
3rd quadrant: $-x - y = 1$ \quad x < 0, y < 0
4th quadrant: $x - y = 1$ \quad x > 0, y < 0
1.1 Exercise 4

Find the domain and range of the function $g$ defined by $g(x) = \sqrt{x^2 - 3x}$.

\[ x^2 - 3x \geq 0 \quad \Rightarrow \quad x(x-3) \geq 0 \]

\[ \begin{array}{cccc}
  + & - & + \\
  0 & 3 \\
\end{array} \]

Domain is $[3, \infty) \cup (-\infty, 0]$

Range is $[0, \infty)$
1.1 Exercise 24

Graph the equation \(|x| + |y| = 1\).

1st quadrant: \(x + y = 1\)
2nd quadrant: \(-x + y = 1\)
3rd quadrant: \(-x - y = 1\)
4th quadrant: \(x - y = 1\)
1.3 Exercise 60

A triangle has sides $a=2$, $b=3$, and angle $C = 40$ degrees. Find the length of side $c$.

\[
\text{law of cosines: } c^2 = a^2 + b^2 - 2ab \cos \theta
\]
\[
c^2 = 4 + 9 - 2(2)(3) \cos 40^\circ
\]
\[
c^2 = 13 - 12 \cos 40^\circ = 3.807
\]
1.5 Exercise 34

Determine how much time is required for an investment to double in value if interest is earned at the rate of 5.75% continuously.

\[ y = Pe^{rt}, \quad y = Pe^{0.0575t} \]

Solve \( e^{0.0575t} = 2 \)

\[ e^{0.0575t} = 2 \]

\[ \ln (e^{0.0575t}) = \ln 2 \]

\[ 0.0575t = \ln 2 \]

\[ t = \frac{\ln 2}{0.0575} = 12.05 \text{ years} \]

\[ t \approx 12 \text{ years} \]

When \( r = 0.10 \),

\[ \approx 6.93 \text{ years} \]