1. Five hundred draws will be made at random with replacement from a box with 4 tickets. The tickets are numbered 2, 3, 5, 6.

(a) Estimate the chance that the sum of the draws will be more than 1,950.

\[
\begin{align*}
\text{sum of tickets:} & \quad 2, 3, 5, 6 \\
\text{ave box:} & \quad 4 \\
\text{SD box:} & \quad 1.58 \\
EV_{\text{sum}} & = 500(4) = 2000 \\
SE_{\text{sum}} & = \sqrt{\frac{500}{1.58}} = 35
\end{align*}
\]

\[
1950 - 2000 = -1.43 \\
\frac{1950 - 2000}{35} = -1.43 \\
85 + 7.5 = 92.5 \text{ (Tiny!)}
\]

(b) Estimate the chance of getting fewer than 90 3's.

The number of 3's is like the sum of 500 draws from the box:

\[
\begin{align*}
\text{sum of tickets:} & \quad 1, 3, 6 \\
\text{ave box:} & \quad 2.5 \\
\text{SD box:} & \quad 4.33 \\
EV_{\text{sum}} & = 500(2.5) = 125 \\
SE_{\text{sum}} & = \sqrt{\frac{500}{4.33}} = 9.68
\end{align*}
\]

\[
90 - 125 = -3.6 \\
\frac{90 - 125}{9.68} = -3.6 \\
\text{z} = -1.15 \text{ (Tiny!)}
\]

2. A box contains 4 tickets with negative numbers and 6 tickets with positive numbers. If we draw 100 tickets at random (with replacement), find the chance we get at least 61 positive numbers.

The number of positives is like the sum of 100 draws from the box:

\[
\begin{align*}
\text{sum of tickets:} & \quad \text{neg, } 6, 1, \text{ pos} \\
\text{ave box:} & \quad 0.6 \\
\text{SD box:} & \quad 1.49 \\
EV_{\text{sum}} & = 100(0.6) = 60 \\
SE_{\text{sum}} & = \sqrt{\frac{100}{1.49}} = 4.9
\end{align*}
\]

\[
\frac{61 - 60}{4.9} = 0.2 \\
16\% \\
100 - 16 = 84\% \\
84\% \approx 4.2\%
\]
3. (20 points) Suppose someone plays a game in which they roll a die. If the die lands 1 or 6, they win $1, otherwise, they lose $1. They plan to play the game 400 times.

(a) How much money do we expect them to win? Give or take how much? (Show your work).

The amount of money they win is like the sum of 400 draws from the box

\[
\begin{array}{c}
\square \quad \square \quad \square \quad \square \quad \square \\
\end{array}
\]

\[
\text{ave}_{\text{box}} = -0.333 \\
\text{SD}_{\text{box}} = 0.943
\]

\[
\begin{align*}
\text{EV}_{\text{sum}} &= 400( -0.333 ) = -133.33 \\
\text{SE}_{\text{sum}} &= \sqrt{400} \cdot 0.943 = 18.86 \\
\end{align*}
\]

\[
\begin{align*}
\text{expect to lose around } &133.33 \\
\text{give or take } &18.86
\end{align*}
\]

(b) Find the chance they lose more than $135.

\[
\begin{align*}
\text{The chance} \\
= \frac{135-133}{18.86} = 0.106 \\
= 46.9\%
\end{align*}
\]

(c) How many times do we expect them to win? Give or take how many? (Show your work).

The number of times they win is like the sum of 400 draws from

\[
\begin{array}{c}
\square \quad \square \quad \square \quad \square \quad \square \\
\end{array}
\]

\[
\text{ave}_{\text{box}} = 0.333 \\
\text{SD}_{\text{box}} = 0.47
\]

\[
\begin{align*}
\text{EV}_{\text{sum}} &= 400(0.333) = 133.2 \\
\text{SE}_{\text{sum}} &= \sqrt{400} \cdot 0.47 = 9.4
\end{align*}
\]

(d) Find the chance they win more than 140 times.

\[
\begin{align*}
\text{The chance} \\
= \frac{140-133.2}{9.4} = 0.72
\end{align*}
\]

4. (10 points) A programmer is working on a new program, COIN, to simulate tossing a coin. As a preliminary test, she sets up the program to do 1 million tosses. The program returns with a count of 502,015 heads. Is something wrong with COIN?

The number of H's is like the sum of 1,000,000 draws from

\[
\begin{array}{c}
\square \quad \square \\
\end{array}
\]

\[
\text{ave}_{\text{box}} = 0.5 \\
\text{SD}_{\text{box}} = 0.5
\]

\[
\begin{align*}
\text{EV}_{\text{sum}} &= 1,000,000(0.5) = 500,000 \\
\text{SE}_{\text{sum}} &= \sqrt{1,000,000} \cdot 0.5 = 500
\end{align*}
\]

502,015 is more than 4 SEs above the expected value so something is wrong with COIN.
5. (10 points) A store has “scratch and win” cards that show a prize of $0, $1, $2, or $3. The chance of getting these prizes is 80%, 10%, 7% and 3% respectively. Find the chance that the store has to pay out more than $20 in prizes for the next 50 customers, assuming the prizes really are random.

The amount they pay is like the sum of 50 draws from the box:

\[ \text{ave}_{\text{box}} = 0.33 \]
\[ \text{SD}_{\text{box}} = 0.74 \]

\[ \text{EV}_{\text{sum}} = 50 \times 0.33 = 16.5 \]
\[ \text{SE}_{\text{sum}} = \sqrt{50} \times 0.74 = 5.2 \]

\[ \frac{20 - 16.5}{5.2} = 0.67 \]

About 26.2%.

6. According to a genetic theory, there is a 15% chance that a randomly selected person from a large population has a given gene. If I take a simple random sample of 1000 people from this population, what is the chance that fewer than 140 have this gene?

The # who have the gene is like the sum of 1000 draws from the box:

\[ \text{ave}_{\text{box}} = 0.15 \]
\[ \text{SD}_{\text{box}} = 0.357 \]

\[ \text{EV}_{\text{sum}} = 1000 \times 0.15 = 150 \]
\[ \text{SE}_{\text{sum}} = \sqrt{1000} \times 0.357 = 11.3 \]

\[ \frac{140 - 150}{11.3} = -0.88 \]

100 - 63% = 37

\[ \frac{3.7}{2} = 18.5\% \]
7. (10 points) A basketball player claims to have an 80% chance of making any free-throw. Assuming her claim is correct, and that her free-throws are independent, find the expected value and the standard error for the number of free-throws she would make if she tried 30 times.

The number of free-throws she would make is like the sum of 30 draws from the box

\[ \text{ave}_{\text{box}} = 0.8 \]
\[ \text{SD}_{\text{box}} = 0.4 \]
\[ \text{EV}_{\text{sum}} = 30(0.8) = 24 \]
\[ \text{SE}_{\text{sum}} = \sqrt{30} \times (0.4) = 2.2 \]

We expect her to make 24 give-or-take 2.2

8. (10 points) A farmer has a flock of 300 ewes. For this breed of ewes, the chance a ewe will bear a single lamb is .75, the chance she will bear twins is .20, and the chance she will bear no lambs is .05. For the upcoming breeding season, find the chance the farmer will get at least 360 lambs from his 300 ewes.

\[ \text{ave}_{\text{box}} = 1.15 \]
\[ \text{SD}_{\text{box}} = 0.48 \]
\[ \text{EV}_{\text{sum}} = 300(1.15) = 345 \]
\[ \text{SE}_{\text{sum}} = \sqrt{300} \times (0.48) = 8.3 \]

\[ 345 \] ? \[ 360 \] ?
\[ \frac{360 - 345}{8.3} = 1.8 \]
About 3.5%?

9. The total amount of gas is like the sum of 100 draws from the box

\[ \text{ave}_{\text{box}} = 9.84 \]
\[ \text{SD}_{\text{box}} = 3.92 \]
\[ \text{EV}_{\text{sum}} = 100(9.84) = 984 \]
\[ \text{SE}_{\text{sum}} = \sqrt{100} \times (3.92) = 39.2 \]

\[ 984 \] 
\[ 1000 \]
\[ z = \frac{67 - 34.5}{39.2} = 0.41 \]