Chance and Probability

Flip a coin:
    Event: “getting heads”
    What is the probability of the event, “getting heads”?
    \[ P(\text{getting heads}) = \frac{1}{2} = 0.5 = 50\% \]
    50\% of the time we expect to get heads in many flips

How do we get 50\% or \( \frac{1}{2} \)?

\[ P(\text{Event}) = \frac{\text{number of successes}}{\text{number of successes and failures}} \]

The successes and failures of an event are the outcomes.

Example: Flip a coin
    Outcomes: Heads, Tails
    \[ P(\text{getting tails}) = \]

Example: Roll a die
    Outcomes: 1, 2, 3, 4, 5, 6
    \[ P(\text{rolling an odd number}) = \]

Example: Flip 2 coins
    Outcomes: HH, HT, TH, TT
    \[ P(\text{rolling at least 1 head}) = \]

Chances, or probabilities, are between 0\% and 100\% (0 and 1).
**Subtraction Rule**
If two events are "opposites" then the
\[ P(\text{event}) = 100\% - P(\text{opposite event}) \]

Example: Flip a coin
Event: Getting heads  Opposite event: Getting tails

Example: Roll a die
Outcomes: 1, 2, 3, 4, 5, 6
Event: rolling an odd number  Opposite event: rolling a number not odd
\[ P(\text{rolling an odd number}) = \]

Event: rolling at least "2"  Opposite event:
\[ P(\text{rolling at least "4"}) = \]

Event: rolling at most "5"  Opposite event:
\[ P(\text{rolling at most "5"}) = \]

Example: Flip 2 coins
Outcomes: HH, HT, TH, TT
Event: rolling at least 1 head  Opposite event:
\[ P(\text{rolling at least 1 head}) = \]

Event: rolling no heads  Opposite event:
\[ P(\text{rolling no heads}) = \]

Event: rolling all tails  Opposite event:
\[ P(\text{rolling all tails}) = \]
Example: Draw one ticket out of a box (randomly)
    Event: drawing a “B”  
    Opposite event: 
    \[ P(\text{drawing a “B”}) = \]

Example: Draw two tickets out of a box
    \textit{With replacement}: after the first draw, the ticket is replaced in the box. (It’s like the box has no \textit{memory} that the first ticket was drawn.)
    
    \textit{Without replacement}: the first tickets is left out of the box when drawing the second time. (It’s like the box has memory of the first ticket being drawn.)
    
    \textit{Given}: The event that occurs first.

Example: Draw two tickets out of a box \textit{with replacement}.
    \[ P(\text{drawing a “B” given an “A” was drawn first}) = \]

Example: Draw two tickets out of a box \textit{without replacement}.
    \[ P(\text{drawing a “B” given an “A” was drawn first}) = \]

\textbf{Conditional Probabilities}

Example: Rolling 2 dice.
    \[ P(\text{getting an “ace” given an “ace” was rolled first}) = \]

Example: Rolling a die and tossing a coin.
    \[ P(\text{getting an “ace” given heads was tossed}) = \]

\textit{Knowing one event has occurred, but it does not change the probability of another event occurring, the two events are \textbf{INDEPENDENT}.}
    
    For example, coins, die, drawing with replacement.
Drawing from a 52-card deck
52 card deck has: 4 suits (hearts, diamonds, spades, clubs)
13 cards in each suit (2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A)
Hearts and Diamonds are Red suits,
Spades and Clubs are Black suits.

Example: Draw 1 card from a 52-card deck
$P(\text{drawing an Ace}) =$

$P(\text{drawing a red card}) =$

$P(\text{drawing spades}) =$

Example: Draw 2 cards from a 52-card deck without replacement.
$P(\text{the first card is an Ace}) =$

$P(\text{the second card is a Jack}) =$

$P(\text{the second card is a Jack given the first card is an Ace}) =$

$P(\text{drawing a heart given a heart was drawn}) =$

Draw 2 cards from a deck, with replacement.
$P(\text{drawing a heart given a heart was drawn}) =$
**Multiplication Rule**

Keyword: AND
Check: INDEPENDENCE

The probability that two events will both occur equals the probability of the first event times the probability of the second event.

\[ P(\text{event one AND event two}) = (P(\text{event one}))(P(\text{event two})) \]

DEPENDENT Multiplication Rule:

\[ P(\text{event one AND event two}) = \\
   (P(\text{event one}))(P(\text{event two GIVEN event one occurred})) \]

Example: Draw 2 cards from a 52-card deck with replacement.

\[ P(\text{drawing an ace and drawing a jack}) = \\
   \text{(check for independence)} \\
   \text{P(drawing an ace) times P(drawing a jack)} \]

Example: Draw 2 cards from a deck without replacement.

\[ P(\text{drawing an ace and drawing a jack}) = \\
   \text{(check for independence)} \\
   \text{P(drawing an ace) times P(drawing a jack given Ace was drawn)} \]

Example: Roll a die and toss a coin.

\[ P(\text{rolling a "6" and getting tails}) = \]
Listing the Outcomes

Toss a coin. Outcomes: H, T 2
Toss 2 coins Outcomes: HH, HT, TH, TT 4 = 2²
Toss 3 coins Outcomes: HHH, HHT, HTH, HTT
THH, THT, TTH, TTT 8 = 2³
Toss 4 coins Number of outcomes =

Roll a die Outcomes: 1, 2, 3, 4, 5, 6 6
Roll 2 dice Outcomes:
(1, 1)  (1, 2)  (1, 3)  (1, 4)  (1, 5)  (1, 6)
(2, 1)  (2, 2)  (2, 3)  (2, 4)  (2, 5)  (2, 6)
(3, 1)  (3, 2)  (3, 3)  (3, 4)  (3, 5)  (3, 6)
(4, 1)  (4, 2)  (4, 3)  (4, 4)  (4, 5)  (4, 6)
(5, 1)  (5, 2)  (5, 3)  (5, 4)  (5, 5)  (5, 6)
(6, 1)  (6, 2)  (6, 3)  (6, 4)  (6, 5)  (6, 6) 36=6²

Roll 3 dice Number of outcomes =

Example: Roll 2 dice
P(rolling a sum of 7) =
P(rolling (1, 6) or (6, 1) or (2, 5) or (5, 2) or (3, 4) or (4, 3))

Mutually Exclusive
If two events can NOT occur simultaneously, then they are mutually exclusive. One excludes the other.

Addition Rule
Keyword: OR
Check: Events must be mutually exclusive
P(event one or event two) = P(event one) + P(event two)
Example: Roll 2 dice
\[ P(\text{rolling a sum of 7}) = \]
\[ P(\text{rolling (1, 6) or (6, 1) or (2, 5) or (5, 2) or (3, 4) or (4, 3)}) = \]
\[ P(\text{rolling a sum of 7 or a sum of 11}) = \]

\[ P(\text{rolling a "1" on first die or rolling a "3" on second die}) \]

Example: Draw from deck without replacement.
\[ P(\text{drawing an ace or drawing a jack}) = \]
\[ P(\text{drawing an ace or drawing a heart}) = \]

p.244, Example 6

Paradox of Chevalier de Mere

Event 1: getting at least one ace on 4 rolls of a die
Opposite Event 1: getting no aces on 4 rolls of a die

Event 2: getting at least one (1, 1) on 24 rolls of a pair of dice.
Opposite Event 2: never getting snake eyes on 24 rolls of a pair.