# Statistics 2000, Section 001, Final (300 Points) 

Wednesday, December 15, 2010

## Part I: Text Answers

## Your Name:

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Question 1: Statistical Inference (50 Points)
A lab scientist is interested in whether lab rats that grow up alone grow as large as lab rats that grow up with other rats around them to play with. He randomly selects ten young rats with approximately the same age and size. Five of these will spend the next 4 months by themselves and the other five rats will each have three other rats to play with during that same time. After 4 months, the scientist measures the abdomen circumference of all the rats (in mm ). The results are shown below:

| Alone group (\#1) | 110 | 123 | 113 | 103 | 120 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Play group (\#2) | 119 | 125 | 131 | 128 | 136 |

The sample means and sample standard deviations are: $\bar{x}_{1}=113.8$ and $s_{1}=7.98$, $\bar{x}_{2}=127.8$ and $s_{2}=6.38$. It is reasonable to believe that rat weights in each group roughly follow a normal distribution. Show your work!

1. (8 Points) What are the hypotheses to test whether the rats grow up to be equally large or whether the playing rats grow up larger? Use the proper mathematical notation and symbols.
2. (4 Points) Briefly justify your choice of a one-sided or two-sided alternative in the previous part.
3. (8 Points) Calculate the test statistic.
4. (6 Points) What are the appropriate degrees of freedom for this test?
5. (6 Points) Determine the P-value.
6. (6 Points) If you use the usual $5 \%$ significance level, should you reject the null hypothesis? Yes / No ? Circle your answer and briefly explain why/why not.
7. (6 Points) If you use the $1 \%$ significance level instead, should you reject the null hypothesis? Yes / No ? Circle your answer and briefly explain why/why not.
8. (6 Points) Give a short summary of your conclusion for the usual $5 \%$ significance level, i.e., how would you explain the result to a person who does not know much about statistics?

Question 2: Regression (40 Points)
Before Thanksgiving, a supermarket ran the following ad regarding their turkey sale:


Frozen Hen Turkeys - \$9 Off
$10-15 \mathrm{lbs}$. average. Price per lb. after $\$ 9$ discount.

|  | 10 lbs. | 11 lbs. | 12 lbs. | 13 lbs. | 14 lbs. | 15 lbs. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Meijer | $\$ 0.29$ | $\$ 0.37$ | $\$ 0.44$ | $\$ 0.50$ | $\$ 0.55$ | $\$ 0.59$ |

Scatterplot of the data:


From a simple random sample of 5 customers who bought a frozen hen turkey, the data in the following table were obtained. A scatterplot of the data is shown on the previous page.

| Turkey Weight | Price per Pound (in \$) | Predicted Price per Pound (in \$) | Residual |
| :---: | :---: | :---: | :---: |
| 10 | 0.29 |  |  |
| 11 | 0.37 |  |  |
| 13 | 0.50 |  |  |
| 14 | 0.55 |  |  |
| 15 | 0.59 |  |  |

The average turkey weight (in this sample of 5) was 12.6000 pounds with a standard deviation of 2.0736 pounds. The average price per pound was $\$ 0.4600$ with a standard deviation of $\$ 0.1261$. The correlation between weight and price per pound was 0.9944 . Work with 4 decimal digits (as above) and show your work!

1. (8 Points) Calculate the regression equation that predicts price per pound from turkey weight.
2. (8 Points) Predict the price per pound, using your regression equation, for turkey weights of $10,11,13,14$, and 15 pounds. Just show one sample calculation below and fill in all results in the table above.
3. (8 Points) Calculate the 5 residuals. Just show one sample calculation below and fill in all results in the table above.
4. (8 Points) Sketch a residual plot that shows turkey weight on the horizontal axis and the residuals on the vertical axis.

5. (8 Points) Based on your residual plot above, is your regression equation a good way to predict the price per pound based on the turkey weight? Yes / No ? Circle your answer and provide some explanation.

Question 3: Probability (40 Points)
For his next road trip, a student places the following 10 CDs into the glove compartment of his car:

- 4 modern rock CDs (Neon Trees, Black Keys, Civil Twilight, Phoenix),
- 3 pop CDs (Katy Perry, Lady Gaga, Ke\$ha),
- 3 American Idol CDs (Lee DeWyze, Kris Allen, David Cook).

On his trip, the student blindly grabs a CD from the glove compartment, listens to it, and places it on the back seat when finished. Then he blindly grabs a second CD from the glove compartment. You should NOT comment on the musical taste of this student, but answer each of the following questions separately. Show your work! As a part of your answer, translate the everyday language into probability statements, using the proper notation, e.g., $P(A), P(A$ and $B), P(A$ or $B), P(A \mid B)$, etc., where $A$ (and $B)$ are the events of interest.

1. (8 Points) What is the chance that he listens to the Katy Perry CD, followed by the Lady Gaga CD? The chance is $\qquad$ \%
2. (8 Points) What is the chance that he will listen to a pop CD as one of his two selections? The chance is $\qquad$ \%
3. (8 Points) What is the chance that he will listen to none of the modern rock CDs? The chance is $\qquad$ \%
4. (8 Points) What is the chance that the second CD will be a pop CD or an American Idol CD? The chance is $\qquad$ \%
5. (8 Points) What is the chance that he will listen to at least one of the American Idol CDs? The chance is $\qquad$ \%

Question 4: Probability and Distributions (50 Points)
A few months later, without changing the CDs, the student takes off for a Spring Break trip to Florida (i.e., he still has 4 modern rock, 3 pop, and 3 American Idol CDs). He plans to listen to 60 CDs on this trip. Therefore, he changes his CD replacement strategy. Instead of placing a finished CD on the back seat, he replaces the CD into the glove compartment, shuffles all CDs in the compartment, and blindly grabs the next CD. He follows this strategy for each CD he listens to. Show your work!

1. (8 Points) What is the chance that he listens to the Katy Perry CD, followed by the Lady Gaga CD as his first 2 selections? The chance is $\qquad$ \%
2. (6 Points) For the remaining parts of this question, the focus will be on the modern rock CDs. We will call listening to a modern rock CD a "success". Using the proper statistical notation, write down the underlying statistical distribution for a random variable $X$ that counts the number of modern rock CDs the student listens to on this road trip.
3. (6 Points) What is the expected number of modern rock CDs he expects to listen to on his trip to Florida.
4. (6 Points) What is the corresponding standard deviation?
5. (6 Points) Can we use a Normal approximation to determine the probability that 20 to 30 of the CDs he will listen to are modern rock CDs? Yes / No. Circle the correct answer and explain why you chose this answer.
6. (10 Points) Independently from your previous answer (i.e., no matter whether you answered yes or no), use a Normal approximation to determine the probability that 20 to 30 of the CDs he will listen to are modern rock CDs. For the purpose of this question part, we believe that the necessary requirements hold.
7. (8 Points) Now, do the same calculation as in the previous part, but use a Normal approximation with continuity correction.

# Statistics 2000, Section 001, Final (300 Points) 

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## Part II: Multiple Choice Questions

Your Name:

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Question 5: Multiple Choice Questions (120 Points)
Mark your answer for each multiple choice question in the table below. There is only one correct answer for each question. Each correct answer is worth 4 points.

| Question | (a) | (b) | (c) | (d) | Question | (a) | (b) | (c) | (d) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 16 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 2 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 17 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 3 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 18 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 4 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 19 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 5 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 24 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 6 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 21 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 7 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 22 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 8 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 23 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 9 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 24 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 10 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 25 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 11 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 26 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 12 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 27 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 13 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 28 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 14 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 29 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 15 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | 30 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |

1. From a population of size 100 (with subjects numbered from 00 to 99 ), we draw a simple random sample of size $n=5$. Which of the following outcomes is more likely?
(a) Subjects $17,29,55,83$, and 97.
(b) Subjects 00, 01, 02, 03, and 04.
(c) Subjects $20,40,60,80$, and 00.
(d) They are all equally likely.
2. A (fair) coin is tossed multiple times. Which of the following outcome sequences is more likely? Note that H represents that the coin lands on heads and T represents that the coin lands on tails.
(a) $\mathrm{H}, \mathrm{H}, \mathrm{H}, \mathrm{H}$.
(b) $\mathrm{H}, \mathrm{T}, \mathrm{T}, \mathrm{H}, \mathrm{T}$.
(c) $\mathrm{T}, \mathrm{T}, \mathrm{T}, \mathrm{T}, \mathrm{T}$.
(d) They are all equally likely.
3. An opinion poll is to be given to a sample of 90 members of a local gym. The members are first divided into men and women, and then a simple random sample of 45 men and a separate simple random sample of 45 women are taken. What is this is an example of?
(a) A block design.
(b) A stratified random sample.
(c) A double-blind simple random sample.
(d) A randomized comparative experiment.
4. A call-in poll conducted by USA Today concluded that Americans love Donald Trump. This conclusion was based on data collected from 7800 calls made by USA Today readers.
(a) What sampling technique is being used?
(b) Simple random sampling.
(c) Stratified random sampling.
(d) Volunteer sampling.
(e) Convenience sampling.
5. An amateur gardener decides to change the variety of tomatoes for this year to see if the yield improves. He put in six plants the previous year and six plants this year using the same part of the garden. The average yield per plant was 11.3 pounds per plant in the previous year and 14.5 pounds per plant using the new variety. What is this an example of?
(a) An experiment.
(b) An observational study, not an experiment.
(c) The elimination of all confounding variables by design, because the gardener used the same part of the garden in both years.
(d) A multistage design, because two years were involved.
6. Seventy-five college students are taking part in a study initiated by a large computer manufacturer. The company is designing a new type of laptop computer and has created prototypes of it with two different keyboard designs. They are also including the current design of the laptop in the experiment. Each of the students was randomly assigned to one of the three types of computers. The students are asked to spend 15 minutes on one of the computers performing several tasks (typing words, numbers, making corrections, etc.). The ease of use of the keyboard was then rated on a five-point scale by having the students fill out a short questionnaire. Which of the following basic principles of statistical design was not used in this experiment?
(a) Control.
(b) Randomization.
(c) Repetition.
(d) Blinding.
7. Let $x=$ the number of people who failed to complete high school and $y=$ the number of infant deaths. According to the 1990 census, those states having an above average value for the variable $x$ tended to have an above average value for $y$. In other words, there was a positive association between $x$ and $y$. What is the most plausible explanation for this association?
(a) Causation: $x$ causes $y$. Thus, programs to keep teens in school will help reduce the number of infant deaths.
(b) Causation: $y$ causes $x$. Thus, programs that reduce infant deaths will ultimately reduce the number of high school dropouts.
(c) Common response: changes in $x$ and $y$ are due to a common response to other variables. For example, states with large populations will have larger numbers of people who fail to complete high school and a larger number of infant deaths.
(d) Accidental: the association between $x$ and $y$ is purely coincidental. It is implausible to believe the observed association could be anything other than accidental.
8. Fill in the blank. A researcher is conducting a study on contact lenses. She finds that in a sample of contact lens-wearing adults, those who wear their contact lenses for longer periods of time have more irritation in their eyes. Those who wear their contact lenses for only a few hours a day have less irritation. However, those who wear their contacts for longer times also tend to spend long hours at the office. As explanations for having more irritation in the eyes, the variables "length of time contacts are worn each day" and "length of time spent at the office per day" are
$\qquad$ variables.
(a) response.
(b) independent.
(c) confounding.
(d) placebo.
9. Which of the following best describes correlation?
(a) Correlation measures the strength of the relationship between two quantitative variables whether or not the relationship is linear.
(b) Correlation measures how much a change in the explanatory variable causes a change in the response variable.
(c) Correlation measures the strength of the relationship between any two variables.
(d) Correlation measures the strength of the linear relationship between two quantitative variables.
10. A researcher at a large company has collected data on the beginning salary $(x)$ and current salary ( $y$ ) of 48 randomly selected employees. The least-squares regression equation for predicting their current salary from their beginning salary is $\hat{y}=-2532.7+2.12 x$.

The current salaries had a mean of $\$ 32,070$ with a standard deviation of $\$ 15,300$. The beginning salaries had a mean of $\$ 16,340$ with a standard deviation of $\$ 5,970$. What is the correlation between current and beginning salary?
(a) $r=0.390$.
(b) $r=0.506$.
(c) $r=0.827$.
(d) Cannot be determined from the information given.
11. A large study was done to compare two treatments for recurring ear infections in young children. A total of 150 subjects were recruited for the study, with 50 subjects assigned at random to treatment 1 and the remainder to treatment 2. As part of the study, various types of demographic information were gathered. The following table classifies the mother's education level into one of four categories for each of the treatment groups.

|  | Not HS grad | HS grad | Some <br> college | College grad |
| :--- | :--- | :--- | :--- | :--- |
| Treatment 1 | 10 | 22 | 12 | 6 |
| Treatment 2 | 23 | 55 | 8 | 14 |

Under the null hypothesis that the distribution of mother's educational level is the same for both treatment groups, the expected number of mothers who are college grads for treatment 2 is
(a) 10.00 .
(b) 13.33 .
(c) 20.00 .
(d) 25.00 .
12. A table has 4 rows and 5 columns. The degrees of freedom for a $\chi^{2}$ test are
(a) 9 .
(b) 10 .
(c) 12 .
(d) 20 .
13. We are interested in comparing the proportions of males and females who think earning a large salary is very important to them. I surveyed 200 of each gender and recorded their answers to the question as "yes" or "no". If I analyze the data with both a z-test for two proportions and a $\chi^{2}$ test of no association, the p-value for the z-test
(a) Will be less than for the $\chi^{2}$ test.
(b) Will be the same as for the $\chi^{2}$ test.
(c) Will be more than for the $\chi^{2}$ test.
(d) It is meaningless to compare these numerical results as they measure two totally different things.
14. A study was conducted to explore the relationship between a child's birth order and his or her chances of becoming a juvenile delinquent. The subjects included a random sample of girls enrolled in public high schools in a large city. Each subject filled out a questionnaire that measured whether or not they had shown delinquent behavior, and their birth order. The data are given in the table below.

|  | Delinquent behavior |  |
| :---: | :---: | :---: |
|  | Yes | No |
| Oldest | 24 | 285 |
| In-between | 29 | 247 |
| Youngest | 35 | 211 |
| Only child | 23 | 70 |

The value of the $\chi^{2}$ statistic for this data set is 21.236 . The P -value is
(a) less than 0.0005 .
(b) between 0.0005 and 0.001 .
(c) between 0.001 and 0.0025 .
(d) between 0.0025 and 0.005 .
15. In the table below we examine the relationship between final grade and the reported hours per week each student said they studied for the course.

|  | A | B | C | D | F | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $<5$ | 2 | 3 | 7 | 4 | 5 | 21 |
| $5-10$ | 3 | 5 | 5 | 4 | 6 | 23 |
| $>10$ | 6 | 5 | 3 | 4 | 2 | 20 |
| Total | 11 | 13 | 15 | 12 | 13 | 64 |

The size of this table is
(a) $7 \times 5$.
(b) $5 \times 7$.
(c) $4 \times 6$.
(d) $3 \times 5$.
16. I have computed an analysis of variance for 3 groups with 10 observations per group. The degrees of freedom for the F test are
(a) 3 and 30 .
(b) 2 and 10 .
(c) 2 and 9 .
(d) 2 and 27 .
17. I have computed an Analysis of Variance (ANOVA) F test and obtained a p-value of 0.001 . This means
(a) All the groups have the same mean.
(b) All the group means are different.
(c) Some of the group means differ from the others.
(d) The result is biased.
18. I have 3 groups for which I want to perform an ANOVA. They have standard deviations $s_{1}=2.5, s_{2}=3.4, s_{3}=6.4$ and the plots of the data indicate all samples are approximately normal with no outliers. Is the ANOVA appropriate?
(a) Yes - because the data are approximately normal distributed.
(b) Yes - because there are no outliers.
(c) No.
(d) I do not have enough information to tell.
19. The heights (in inches) of adult males in the U.S. are believed to be normally distributed with mean $\mu$. The average height of a random sample of 25 American adult males is found to be $\bar{x}=69.72$ inches and the standard deviation of the 25 heights is found to be $s=4.15$. A $90 \%$ confidence interval for $\mu$ is
(a) $69.72 \pm 1.06$.
(b) $69.72 \pm 1.09$.
(c) $69.72 \pm 1.37$.
(d) $69.72 \pm 1.42$.
20. It has been claimed that women live longer than men; however, men tend to be older than their wives. Ages of 16 deceased husbands and wives from England were obtained via a simple random sample of death records. These data should be analyzed with a
(a) Two-sample t-test.
(b) Paired samples t-test
(c) Two-sample z-test.
(d) $\chi^{2}$ test.
21. The mathematical expression 5 ! (read 5 factorial) numerically results in
(a) 1 .
(b) 5 .
(c) 24 .
(d) 120 .
22. The mathematical expression $\binom{6}{4}$ (read 6 choose 4) numerically results in
(a) 1.5 .
(b) 10 .
(c) 15 .
(d) 30 .
23. The time for students to complete a standardized placement exam given to college freshman has a normal distribution with a mean of 45 minutes and a standard deviation of 10 minutes. If students are given one hour to complete the exam, the proportion of students who will complete the exam is about
(a) 0.17 .
(b) 0.40 .
(c) 0.60 .
(d) 0.93 .
24. The scores on a university examination are normally distributed with a mean of 65 and a standard deviation of 11 . If the top $8 \%$ of students are given A's, what is the lowest mark that a student can have and still be awarded an A?
(a) 49.5 .
(b) 76.1 .
(c) 80.5 .
(d) 84.2 .
25. In a particular game, a ball is randomly chosen from a box that contains 2 red balls, 2 green ball, and 6 blue balls. If a red ball is selected you win $\$ 2$, if a green ball is selected you win $\$ 4$, and if a blue ball is selected you win nothing. Let $X$ be the amount that you win. The expected value of $X$ is
(a) $\$ 1.20$.
(b) $\$ 1.50$.
(c) $\$ 2.00$.
(d) $\$ 3.00$.
26. You decide to test a friend for ESP using a standard deck of 52 playing cards. Such a deck contains 13 spades, 13 hearts, 13 diamonds, and 13 clubs. You shuffle the deck, select a card at random, and ask your friend to tell you whether the card is a spade, heart, diamond, or club. After the guess you return the card to the deck, shuffle the cards, and repeat the above. You do this a total of 80 times. Let $X$ be the number of correct guesses by your friend in the 80 trials. The standard deviation of $X$ is
(a) about 0.05.
(b) about 3.87 .
(c) about 15.65 .
(d) about 225.87.
27. Incomes in a certain town are strongly right skewed with mean $\$ 40,000$ and standard deviation $\$ 8,000$. A random sample of 5 households is taken. What is the probability the average of the sample is more than $\$ 42,000$ ?
(a) 0.29 .
(b) 0.40 .
(c) 0.60 .
(d) Cannot say.

## Use the following to answer questions 28, 29, and 30:

In statistics, we usually refer to $x_{1}$ as the first observation, $x_{2}$ as the second observation, etc., and $x_{n}$ as the final observation when we write down our observations in the order they were obtained (where $n$ represents the total number of observations).

Often, we prefer to work with data that are sorted from smallest to largest, e.g., when calculating the median, we need the data to be sorted. Obviously, we can simply reorder any given list of numbers. However, we often use the notation $x_{(1)}$ to refer to the smallest observation, $x_{(2)}$ to refer to the 2nd smallest observation, etc., and $x_{(n)}$ to refer to the largest observation.
28. For $x_{1}=3, x_{2}=-7, x_{3}=-5, x_{4}=-2, x_{5}=5, x_{6}=7$, and $n=6$, the sum

$$
\sum_{i=2}^{n-1} x_{i}=
$$

equals
(a) -30 .
(b) -12 .
(c) -9 .
(d) 1 .
29. For $x_{1}=3, x_{2}=-7, x_{3}=-5, x_{4}=-2, x_{5}=5, x_{6}=7$, and $n=6$, the sum

$$
\sum_{i=2}^{n-1} x_{(i)}=
$$

equals
(a) -30 .
(b) -12 .
(c) -9 .
(d) 1 .
30. For $x_{1}=3, x_{2}=-7, x_{3}=-5, x_{4}=-2, x_{5}=5, x_{6}=7$, and $n=6$, the sum

$$
\sum_{i=1}^{n} x_{(3)}=
$$

equals
(a) -30 .
(b) -12 .
(c) -9 .
(d) 1 .

## TABLES

You can remove the following 2 sheets of tables from the exam. No need to turn in these tables once they have been removed.


| Standard normal probabilities |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| -3.4 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0002 |
| -3.3 | . 0005 | . 0005 | . 0005 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0003 |
| -3.2 | . 0007 | . 0007 | . 0006 | . 0006 | . 0006 | . 0006 | . 0006 | . 0005 | . 0005 | . 0005 |
| -3.1 | . 0010 | . 0009 | . 0009 | . 0009 | . 0008 | . 0008 | . 0008 | . 0008 | . 0007 | . 0007 |
| -3.0 | . 0013 | . 0013 | . 0013 | . 0012 | . 0012 | . 0011 | . 0011 | . 0011 | . 0010 | . 0010 |
| -2.9 | . 0019 | . 0018 | . 0018 | . 0017 | . 0016 | . 0016 | . 0015 | . 0015 | . 0014 | . 0014 |
| -2.8 | . 0026 | . 0025 | . 0024 | . 0023 | . 0023 | . 0022 | . 0021 | . 0021 | . 0020 | . 0019 |
| -2.7 | . 0035 | . 0034 | . 0033 | . 0032 | . 0031 | . 0030 | . 0029 | . 0028 | . 0027 | . 0026 |
| -2.6 | . 0047 | . 0045 | . 0044 | . 0043 | . 0041 | . 0040 | . 0039 | . 0038 | . 0037 | . 0036 |
| -2.5 | . 0062 | . 0060 | . 0059 | . 0057 | . 0055 | . 0054 | . 0052 | . 0051 | . 0049 | . 0048 |
| -2.4 | . 0082 | . 0080 | . 0078 | . 0075 | . 0073 | . 0071 | . 0069 | . 0068 | . 0066 | . 0064 |
| -2.3 | . 0107 | . 0104 | . 0102 | . 0099 | . 0096 | . 0094 | . 0091 | . 0089 | . 0087 | . 0084 |
| -2.2 | . 0139 | . 0136 | . 0132 | . 0129 | . 0125 | . 0122 | . 0119 | . 0116 | . 0113 | . 0110 |
| -2.1 | . 0179 | . 0174 | . 0170 | . 0166 | . 0162 | . 0158 | . 0154 | . 0150 | . 0146 | . 0143 |
| -2.0 | . 0228 | . 0222 | . 0217 | . 0212 | . 0207 | . 0202 | . 0197 | . 0192 | . 0188 | . 0183 |
| -1.9 | . 0287 | . 0281 | . 0274 | . 0268 | . 0262 | . 0256 | . 0250 | . 0244 | . 0239 | . 0233 |
| -1.8 | . 0359 | . 0351 | . 0344 | . 0336 | . 0329 | . 0322 | . 0314 | . 0307 | . 0301 | . 0294 |
| -1.7 | . 0446 | . 0436 | . 0427 | . 0418 | . 0409 | . 0401 | . 0392 | . 0384 | . 0375 | . 0367 |
| -1.6 | . 0548 | . 0537 | . 0526 | . 0516 | . 0505 | . 0495 | . 0485 | . 0475 | . 0465 | . 0455 |
| -1.5 | . 0668 | . 0655 | . 0643 | . 0630 | . 0618 | . 0606 | . 0594 | . 0582 | . 0571 | . 0559 |
| -1.4 | . 0808 | . 0793 | . 0778 | . 0764 | . 0749 | . 0735 | . 0721 | . 0708 | . 0694 | . 0681 |
| -1.3 | . 0968 | . 0951 | . 0934 | . 0918 | . 0901 | . 0885 | . 0869 | . 0853 | . 0838 | . 0823 |
| -1.2 | . 1151 | . 1131 | . 1112 | . 1093 | . 1075 | . 1056 | . 1038 | . 1020 | . 1003 | . 0985 |
| -1.1 | . 1357 | . 1335 | . 1314 | . 1292 | . 1271 | . 1251 | . 1230 | . 1210 | . 1190 | . 1170 |
| -1.0 | . 1587 | . 1562 | . 1539 | . 1515 | . 1492 | . 1469 | . 1446 | . 1423 | . 1401 | . 1379 |
| -0.9 | . 1841 | . 1814 | . 1788 | . 1762 | . 1736 | . 1711 | . 1685 | . 1660 | . 1635 | . 1611 |
| -0.8 | . 2119 | . 2090 | . 2061 | . 2033 | . 2005 | . 1977 | . 1949 | . 1922 | . 1894 | . 1867 |
| -0.7 | . 2420 | . 2389 | . 2358 | . 2327 | . 2296 | . 2266 | . 2236 | . 2206 | . 2177 | . 2148 |
| -0.6 | . 2743 | . 2709 | . 2676 | . 2643 | . 2611 | . 2578 | . 2546 | . 2514 | . 2483 | . 2451 |
| -0.5 | . 3085 | . 3050 | . 3015 | . 2981 | . 2946 | . 2912 | . 2877 | . 2843 | . 2810 | . 2776 |
| -0.4 | . 3446 | . 3409 | . 3372 | . 3336 | . 3300 | . 3264 | . 3228 | . 3192 | . 3156 | . 3121 |
| -0.3 | . 3821 | . 3783 | . 3745 | . 3707 | . 3669 | . 3632 | . 3594 | . 3557 | . 3520 | . 3483 |
| -0.2 | . 4207 | . 4168 | . 4129 | . 4090 | . 4052 | . 4013 | . 3974 | . 3936 | . 3897 | . 3859 |
| -0.1 | . 4602 | . 4562 | . 4522 | . 4483 | . 4443 | . 4404 | . 4364 | . 4325 | . 4286 | . 4247 |
| 0.0 | . 5000 | . 4960 | . 4920 | . 4880 | . 4840 | . 4801 | . 4761 | . 4721 | . 4681 | . 4641 |

Table entry for $z$ is the area under the
standard normal curve to the left of $z$.


TABLE A

| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | . 5040 | . 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 0.3 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | . 6915 | . 6950 | . 6985 | . 7019 | . 7054 | . 7088 | . 7123 | . 7157 | . 7190 | . 7224 |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | . 7549 |
| 0.7 | . 7580 | . 7611 | . 7642 | . 7673 | . 7704 | . 7734 | . 7764 | . 7794 | . 7823 | . 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | . 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
| 1.5 | . 9332 | . 9345 | . 9357 | . 9370 | . 9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | . 9616 | . 9625 | . 9633 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
| 2.5 | . 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | . 9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |

Tables
T-11


TABLE D
$t$ distribution critical values

|  |  |  |  |  |  |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |



TABLE F
$\chi^{2}$ distribution critical values

|  | Tail probability $p$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| df | . 25 | . 20 | . 15 | . 10 | . 05 | . 025 | . 02 | . 01 | . 005 | . 0025 | . 001 | . 0005 |
| 1 | 1.32 | 1.64 | 2.07 | 2.71 | 3.84 | 5.02 | 5.41 | 6.63 | 7.88 | 9.14 | 10.83 | 12.12 |
| 2 | 2.77 | 3.22 | 3.79 | 4.61 | 5.99 | 7.38 | 7.82 | 9.21 | 10.60 | 11.98 | 13.82 | 15.20 |
| 3 | 4.11 | 4.64 | 5.32 | 6.25 | 7.81 | 9.35 | 9.84 | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4 | 5.39 | 5.99 | 6.74 | 7.78 | 9.49 | 11.14 | 11.67 | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5 | 6.63 | 7.29 | 8.12 | 9.24 | 11.07 | 12.83 | 13.39 | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6 | 7.84 | 8.56 | 9.45 | 10.64 | 12.59 | 14.45 | 15.03 | 16.81 | 18.55 | 20.25 | 22.46 | 24.10 |
| 7 | 9.04 | 9.80 | 10.75 | 12.02 | 14.07 | 16.01 | 16.62 | 18.48 | 20.28 | 22.04 | 24.32 | 26.02 |
| 8 | 10.22 | 11.03 | 12.03 | 13.36 | 15.51 | 17.53 | 18.17 | 20.09 | 21.95 | 23.77 | 26.12 | 27.87 |
| 9 | 11.39 | 12.24 | 13.29 | 14.68 | 16.92 | 19.02 | 19.68 | 21.67 | 23.59 | 25.46 | 27.88 | 29.67 |
| 10 | 12.55 | 13.44 | 14.53 | 15.99 | 18.31 | 20.48 | 21.16 | 23.21 | 25.19 | 27.11 | 29.59 | 31.42 |
| 11 | 13.70 | 14.63 | 15.77 | 17.28 | 19.68 | 21.92 | 22.62 | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12 | 14.85 | 15.81 | 16.99 | 18.55 | 21.03 | 23.34 | 24.05 | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13 | 15.98 | 16.98 | 18.20 | 19.81 | 22.36 | 24.74 | 25.47 | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14 | 17.12 | 18.15 | 19.41 | 21.06 | 23.68 | 26.12 | 26.87 | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15 | 18.25 | 19.31 | 20.60 | 22.31 | 25.00 | 27.49 | 28.26 | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16 | 19.37 | 20.47 | 21.79 | 23.54 | 26.30 | 28.85 | 29.63 | 32.00 | 34.27 | 36.46 | 39.25 | 41.31 |
| 17 | 20.49 | 21.61 | 22.98 | 24.77 | 27.59 | 30.19 | 31.00 | 33.41 | 35.72 | 37.95 | 40.79 | 42.88 |
| 18 | 21.60 | 22.76 | 24.16 | 25.99 | 28.87 | 31.53 | 32.35 | 34.81 | 37.16 | 39.42 | 42.31 | 44.43 |
| 19 | 22.72 | 23.90 | 25.33 | 27.20 | 30.14 | 32.85 | 33.69 | 36.19 | 38.58 | 40.88 | 43.82 | 45.97 |
| 20 | 23.83 | 25.04 | 26.50 | 28.41 | 31.41 | 34.17 | 35.02 | 37.57 | 40.00 | 42.34 | 45.31 | 47.50 |
| 21 | 24.93 | 26.17 | 27.66 | 29.62 | 32.67 | 35.48 | 36.34 | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22 | 26.04 | 27.30 | 28.82 | 30.81 | 33.92 | 36.78 | 37.66 | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23 | 27.14 | 28.43 | 29.98 | 32.01 | 35.17 | 38.08 | 38.97 | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24 | 28.24 | 29.55 | 31.13 | 33.20 | 36.42 | 39.36 | 40.27 | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |
| 25 | 29.34 | 30.68 | 32.28 | 34.38 | 37.65 | 40.65 | 41.57 | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26 | 30.43 | 31.79 | 33.43 | 35.56 | 38.89 | 41.92 | 42.86 | 45.64 | 48.29 | 50.83 | 54.05 | 56.41 |
| 27 | 31.53 | 32.91 | 34.57 | 36.74 | 40.11 | 43.19 | 44.14 | 46.96 | 49.64 | 52.22 | 55.48 | 57.86 |
| 28 | 32.62 | 34.03 | 35.71 | 37.92 | 41.34 | 44.46 | 45.42 | 48.28 | 50.99 | 53.59 | 56.89 | 59.30 |
| 29 | 33.71 | 35.14 | 36.85 | 39.09 | 42.56 | 45.72 | 46.69 | 49.59 | 52.34 | 54.97 | 58.30 | 60.73 |
| 30 | 34.80 | 36.25 | 37.99 | 40.26 | 43.77 | 46.98 | 47.96 | 50.89 | 53.67 | 56.33 | 59.70 | 62.16 |
| 40 | 45.62 | 47.27 | 49.24 | 51.81 | 55.76 | 59.34 | 60.44 | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50 | 56.33 | 58.16 | 60.35 | 63.17 | 67.50 | 71.42 | 72.61 | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60 | 66.98 | 68.97 | 71.34 | 74.40 | 79.08 | 83.30 | 84.58 | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80 | 88.13 | 90.41 | 93.11 | 96.58 | 101.9 | 106.6 | 108.1 | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1 | 111.7 | 114.7 | 118.5 | 124.3 | 129.6 | 131.1 | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |

