Statistics 2000, Section 001, Final (300 Points)

Wednesday, December 9, 2009

Your Name: ____________________

Question 1: Miscellaneous Short Questions (**40 Points**)

Please answer the following short questions. **Show your work!**

1. (**8 Points**) Universal blood donors: People with type O–negative blood are universal donors. That is, any patient can receive a transfusion of O–negative blood. About 12% of a particular population have O–negative blood. If 6 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor?

2. (**8 Points**) A coin is slightly bent, and as a result the probability of a head is 0.55. Suppose that you toss the coin 5 times. Use the binomial formula to find the probability of 4 or more heads.

3. (**16 Points**) Portfolio analysis. The means, standard deviations, and correlations for the annual returns from three Fidelity mutual funds for the 10 years ending in February 2004 are listed below. The following subscripts are defined as

$W =$ annual return on 500 Index Fund
$X =$ annual return on Investment Grade Bond Fund
$Y =$ annual return on Diversified International Fund

These subscripts (W, X, and Y) show which random variable/pair of random variables a numerical characteristic, indicated below, refers to.

$$\mu_W = 10.12\%, \qquad \sigma_W = 17.46\%$$
$$\mu_X = 6.46\%, \qquad \sigma_X = 4.18\%$$
$$\mu_Y = 11.10\%, \qquad \sigma_Y = 15.62\%$$

Correlations:

$$\rho_{WX} = -0.22, \quad \rho_{WY} = 0.56, \quad \rho_{XY} = -0.12$$

Assume that a portfolio contains 40% Investment Grade Bond Fund (X) and 60% 500 Index Fund (W) stocks. Calculate the mean and standard deviation of the returns for this portfolio.

4. (**8 Points**) A highly nervous basketball player has two free throw attempts. The player will make a hit with a probability of 0.7 in the first attempt. The outcome of the second attempt highly depends on the outcome of the first attempt: If the first attempt was a hit, the player will also make a hit with a probability of 0.7 in the second attempt. However, if the first attempt was a miss, the player will also miss in the second attempt with a probability of 0.8.

First, draw a tree diagram for the verbal description given above.

Let X be a random variable that represents the number of baskets made out of the two shots. Then, use your tree diagram from above and fill in the following table. Don't forget to check whether the distribution in your table is legitimate (recall which conditions must hold for probabilities).

Value of X	0	1	2
Probability			

Question 2: Confidence Intervals and Hypothesis Testing I (**80 Points**)

A friend who hears that you are taking a statistics course asks for help with a chemistry lab report. She has made four independent measurements of the specific gravity of a compound. The results are:

$$3.82, \quad 3.93, \quad 3.67, \quad 3.78$$

The lab manual says that repeated measurements will vary according to a Normal distribution with standard deviation $\sigma = 0.15$ (this standard deviation shows how precise the measurement process is). The mean μ of the distribution of measurements is the true specific gravity.

Show your work!

Part 1: The lab manual asks for a 95% confidence interval for the true specific gravity.

1. **(6 Points)** First, what would be a good estimator for the unknown population parameter? Indicate the name of this estimator (be specific!), the proper mathematical symbol, and the numerical values.

2. **(7 Points)** Calculate the margin of error for your 95% confidence interval.

3. **(6 Points)** Now, construct a 95% confidence interval for the true specific gravity.

4. **(6 Points)** Can we be certain that the true population parameter falls in this interval? **Yes / No ?** Circle your answer.

5. **(6 Points)** What critical value from the Normal table would you use if you wanted 80% confidence instead of 95% confidence? You do not have to calculate this interval — just indicate the value from the Normal table that is needed.

6. **(6 Points)** Would the 80% confidence interval be **(i) wider** or **(ii) narrower** than your 95% confidence interval from 3. above? [Do NOT actually compute the 80% confidence interval — just circle the correct answer.]

7. **(7 Points)** How many measurements would be needed to estimate the true mean μ within $\pm$ 0.001 with 95% confidence, i.e., what would be the necessary sample size to obtain this margin of error?

Part 2: The lab manual also asks whether the data show convincingly that the true specific gravity is less than 3.9. To answer this question you decide to carry out a test of significance.

8. **(6 Points)** First, state the appropriate null and alternative hypotheses used to answer this question. Use the proper mathematical notation and symbols.

   H_0 :

   H_a :

9. **(6 Points)** Calculate the test statistic.

10. **(6 Points)** Determine the P–value.

11. **(6 Points)** If you use the usual 5% significance level, should you reject the null hypothesis? **Yes / No ?** Circle your answer and explain why/why not.

12. **(6 Points)** If you use the 10% significance level instead, should you reject the null hypothesis? **Yes / No ?** Circle your answer and explain why/why not.

13. **(6 Points)** State your conclusion in terms of the context of the problem for the usual 5% significance level, i.e., how would you explain the result to your friend?

Question 3: Hypothesis Testing II (**60 Points**)

I am a little skeptical of the claim that the average weight of a particular type of tomato is 5.0 ounces. I think it might be somewhat greater than 5.0 ounces. I select 3 tomatoes at random and record their weights:

$$6.0, \quad 4.8, \quad 7.2$$

Then, I create a normal quantile plot. This plot only shows some minor wiggles without significant curvature or outliers, i.e., we can assume that the weights follow a normal distribution. Our main question now is: Is there evidence based on the significance level $\alpha = 0.05$ that the average weight of this type of tomato is greater than 5.0 ounces?

Show your work!

1. **(10 Points)** State which test of significance are you going to use (e.g., z, t, chi–square, one–sided or two–sided, one–sample or two–sample, etc.). Clearly indicate the crucial assumption(s) allowing you to use the test you chose.

2. **(8 Points)** Clearly state the null and alternative hypotheses. Use the proper mathematical notation and symbols.

   H_0 :

   H_a :

3. **(6 Points)** Calculate the sample mean $\overline{x}$ and the sample standard deviation s. You can use the statistical mode of your calculator or you can do this by hand. No need to write down any formulas or intermediate results for this step. Just indicate the final results:

   $\overline{x} =$

   $s =$

4. **(8 Points)** Calculate the appropriate test statistic.

5. **(8 Points)** Determine the P–value.

6. **(6 Points)** If you use the usual 5% significance level, should you reject the null hypothesis? **Yes / No ?** Circle your answer and explain why/why not.

7. **(6 Points)** State your conclusion.

8. **(8 Points)** If it was believed that the weights of tomatoes do not follow the normal curve, would your significance test still be valid? **Yes / No ?** Circle your answer and explain briefly.

Question 4: Multiple Choice Questions **(120 Points)**

Mark your answer for each multiple choice question in the table below. There is only one correct answer for each question. Each correct answer is worth 4 points.

Question	(a)	(b)	(c)	(d)	Question	(a)	(b)	(c)	(d)
1	○	○	○	○	16	○	○	○	○
2	○	○	○	○	17	○	○	○	○
3	○	○	○	○	18	○	○	○	○
4	○	○	○	○	19	○	○	○	○
5	○	○	○	○	20	○	○	○	○
6	○	○	○	○	21	○	○	○	○
7	○	○	○	○	22	○	○	○	○
8	○	○	○	○	23	○	○	○	○
9	○	○	○	○	24	○	○	○	○
10	○	○	○	○	25	○	○	○	○
11	○	○	○	○	26	○	○	○	○
12	○	○	○	○	27	○	○	○	○
13	○	○	○	○	28	○	○	○	○
14	○	○	○	○	29	○	○	○	○
15	○	○	○	○	30	○	○	○	○

1. The time for students to complete a standardized placement exam given to college freshman has a normal distribution with a mean of 62 minutes and a standard deviation of 8 minutes. If students are given one hour to complete the exam, the proportion of students who will complete the exam is about

   (a) 0.25.
   (b) 0.40.
   (c) 0.60.
   (d) 0.75.

2. The scores on a university examination are normally distributed with a mean of 62 and a standard deviation of 11. If the top 10% of students are given A's, what is the lowest mark that a student can have and still be awarded an A?

   (a) 1.28.
   (b) 63.28.
   (c) 70.97.
   (d) 76.08.

3. You receive a fax with six bids (in millions of dollars)

$$2.2,\ 1.3,\ 1.9,\ 1.2,\ 2.4,\ \text{and}\ x,$$

   where x is some number that is too blurry to read. Without knowing what x is, the median

   (a) is 1.8.
   (b) is 1.9.
   (c) must be between 1.3 and 2.2.
   (d) could be any number between 1.2 and 2.4.

4. Suppose you want to know how salaries (in thousands of dollars per year) are related to years of experience for your chosen career. You collect data for a random sample of 100 people with this type of job who have had from 0 to 20 years of experience. The relationship is linear and you determine the regression equation to be: salary = 45.59 + 0.798 years. For this equation, the number 45.59 indicates

   (a) the y–intercept is meaningless because of extrapolation.
   (b) an individual with no experience in this career will earn $45.59 per hour.
   (c) an individual with no experience in this career will earn $45,590 per year.
   (d) this cannot be answered before we know the value of x (i.e., years).

5. Foresters use regression to predict the volume of timber in a tree using easily measured quantities such as diameter. Let y be the volume of timber in cubic feet and x be the diameter in feet (measured at 3 feet above ground level). One set of data gives $y = -30 + 60 \cdot x$. The predicted volume for a huge redwood tree of 18 feet diameter is

   (a) 1050 cubic feet.

   (b) 540 cubic feet.

   (c) 90 cubic feet.

   (d) 60 cubic feet.

6. In the table below we examine the relationship between final grade and the reported hours per week each student said they studied for the course.

	A	B	C	D	F	Total
< 5	2	3	7	4	5	21
$5 - 10$	3	5	5	4	6	23
> 10	6	5	3	4	2	20
Total	11	13	15	12	13	64

   The probability that a student earned an A if they studied more than 10 hours per week is

   (a) 11/64.

   (b) 6/20.

   (c) 20/64.

   (d) 6/64.

7. The regression line to predict average exam grade from hours of study is $y = 15 + 5.6 \cdot x$. The slope of the regression line indicates

   (a) for any student, an extra hour of study increases the grade 5.6 points.

   (b) on average, an extra hour of study will increase the grade 5.6 points.

   (c) on average, an extra hour of study will increase the grade 15 points.

   (d) on average, an extra hour of study will increase the grade 20.6 points.

8. Suppose you are going to roll a die 60 times and record $\hat{p}$, the proportion of times that a 1 or a 2 is showing. The sampling distribution of $\hat{p}$ should be centered about

   (a) 1/6.

   (b) 1/3.

   (c) 1/2.

   (d) 20.

9. Which of the following measures are not affected by outliers?
   (a) The mean.
   (b) The standard deviation.
   (c) The IQR.
   (d) None (all are effected by outliers).

10. A simple random sample of 1000 Americans found that 61% were satisfied with the service provided by the dealer from which they bought their car. A simple random sample of 1000 Canadians found that 58% were satisfied with the service provided by the dealer from which they bought their car. The sampling variability associated with these statistics is
    (a) about the same.
    (b) much smaller for the sample of Canadians since the population of Canada is smaller than that of the United States, hence the sample is a larger proportion of the population.
    (c) much larger for the Canadians since Canada has a lower population density than the United States, hence Canadians tend to live farther apart which always increases sampling variability.
    (d) exactly $61/58 = 1.05172...$.

11. In a particular game, a ball is randomly chosen from a box that contains 3 red balls, 1 green ball, and 6 blue balls. If a red ball is selected you win $2, if a green ball is selected you win $4, and if a blue ball is selected you win nothing. Let X be the amount that you win. The expected value of X is
    (a) nothing ($0).
    (b) $1.
    (c) $2.
    (d) $3.

12. A table has 4 rows and 5 columns. The degrees of freedom for a χ^2 test are
    (a) 9.
    (b) 10.
    (c) 12.
    (d) 20.

13. In a controversial election district, **73%** of registered voters are Democrat. A random survey of 500 voters had **68%** Democrats. Are the bold numbers parameters or statistics?

    (a) Both are statistics.

    (b) 73% is a parameter and 68% is a statistic.

    (c) 73% is a statistic and 68% is a parameter.

    (d) Neither is a statistic.

14. The weight of a medium–sized orange selected at random from a large bin of oranges at the local supermarket is a random variable with mean $\mu = 12$ ounces and standard deviation $\sigma = 1.20$ ounces. Suppose we independently pick two oranges at random from the bin. The difference in the weights of the two oranges selected (the weight of first orange minus the weight of the second orange) is a random variable with standard deviation

    (a) of about 0.00 ounces.

    (b) of about 1.20 ounces.

    (c) of about 1.70 ounces.

    (d) of about 2.88 ounces.

15. On a certain airline, the chance the early flight from Atlanta to Chicago is full is 0.8. The chance the late flight is full is 0.7. The chance both flights are full is 0.6. Are the two flights being full independent events?

    (a) We do not have enough information.

    (b) There must be something wrong with this question as the probabilities sum up to 2.1 — and this is impossible!

    (c) Yes.

    (d) No.

16. You decide to test a friend for ESP using a standard deck of 52 playing cards. Such a deck contains 13 spades, 13 hearts, 13 diamonds, and 13 clubs. You shuffle the deck, select a card at random, and ask your friend to tell you whether the card is a spade, heart, diamond, or club. After the guess you return the card to the deck, shuffle the cards, and repeat the above. You do this a total of 100 times. Let X be the number of correct guesses by your friend in the 100 trials. The standard deviation of X is

    (a) about 25.

    (b) about 18.75.

    (c) about 4.33.

    (d) about 0.433.

17. There are twenty multiple–choice questions on an exam, each having responses a, b, c, or d. Each question is worth 5 points and only one response per question is correct. Suppose a student guesses the answer to each question, and her guesses from question to question are independent. The student's mean score on the exam should be

    (a) 3.75 points.
    (b) 5 points.
    (c) 25 points.
    (d) 50 points.

18. As part of a promotion for a new type of cracker, free trial samples are offered to shoppers in a local supermarket. The probability that a shopper will buy a packet of crackers after tasting the free sample is 0.2. Different shoppers can be regarded as independent trials. If $\hat{p}$ is the proportion of the next 100 shoppers that buy a packet of the crackers after tasting a free sample, then $\hat{p}$ has approximately a Normal distribution with mean $\mu = 0.2$ and

    (a) standard deviation $\sigma = 4$.
    (b) standard deviation $\sigma = 16$.
    (c) standard deviation $\sigma = 0.0016$.
    (d) standard deviation $\sigma = 0.04$.

19. Incomes in a certain town are strongly right skewed with mean $36,000 and standard deviation $7,000. A random sample of 10 households is taken. What is the probability the average of the sample is more than $38,000?

    (a) 0.3875
    (b) 0.1831.
    (c) 0.8169.
    (d) Cannot say.

20. The value of t^* for a 95% confidence interval when there were 10 pieces of data is

    (a) 2.262.
    (b) 2.228.
    (c) 1.833.
    (d) 1.812.

21. For small samples, confidence intervals based on the t–distribution (i.e., "t–intervals") are __________ confidence intervals based on the Normal distribution (i.e., "z–intervals") based on the same data set.

   (a) narrower than
   (b) the same as
   (c) wider than
   (d) exactly twice as wide as

22. You are thinking of using a t–procedure to construct a 95% confidence interval for the mean of a population. You suspect the distribution of the population is not normal and may be skewed. Which of the following statements is correct?

   (a) You should not use the t–procedure since the population does not have a normal distribution.
   (b) You may use the t–procedure provided your sample is small, say at most 10.
   (c) You may use the t–procedure provided your sample size is large, say at least 50.
   (d) You may always use the t–procedure since it is robust to non–normality.

23. Suppose that a random sample of 41 state college students is asked to measure the length of their right foot in centimeters. A 95% confidence interval for the mean foot length for students at this university turns out to be (21.709,25.091). Which of the following is true?

   (a) The sample mean was 23.4 cm.
   (b) The margin of error is 3.382.
   (c) If the confidence level is changed to 90% we will get a wider interval.
   (d) Foot length is not normally distributed so we are not allowed to calculate a conference interval here.

24. The diameter of a spindle in a small motor is supposed to be 5 mm. If the spindle is either too small or too large, the motor will not perform properly. Assume the population of spindles produced by a manufacturer is normally distributed with unknown mean μ and standard deviation $\sigma = 0.02$ mm. The manufacturer measures a SRS of 16 spindles produced by his company and finds their mean diameter to be $\overline{x} = 5.01$ mm. The P–value of a test of the hypotheses $H_0 : \mu = 5, \quad H_a : \mu \neq 5$ is

   (a) 0.0228.
   (b) 0.0456.
   (c) less than 0.0005 (but bigger than 0.0001).
   (d) less than 0.0001.

25. Suppose the average Math SAT score for all students taking the exam this year is 480 with standard deviation 100. Assume the distribution of scores is normal. The senator of a particular state notices that the mean score for students in his state who took the Math SAT is 500. His state recently adopted a new mathematics curriculum and he wonders if the improved scores are evidence that the new curriculum has been successful. Since over 10,000 students in his state took the Math SAT, he can show that the P–value for testing whether the mean score in his state is more than the national average of 480 is less than 0.0001. We may correctly conclude that

    (a) there is strong statistical evidence that the new curriculum has improved Math SAT scores in his state.
    (b) although the results are statistically significant, they are not practically significant, because an increase of 20 points is fairly small.
    (c) these results are not good evidence that the new curriculum has improved Math SAT scores.
    (d) the observed difference is most likely due to chance error.

26. An instructor wanted to construct a confidence interval for the mean GPA of the students in his class. He used the campus records system to obtain their GPA's and computed the 95% interval as (2.32, 2.87). If he wants to use this interval to describe the students in his class

    (a) he is 95% confident the interval contains the real average GPA.
    (b) there is a 95% chance that the interval contains the real average GPA.
    (c) there is a 5% chance the interval is wrong.
    (d) he didn't need an interval after all.

27. A random sample of 85 students in Chicago city high schools take a course designed to improve SAT scores. Based on these students, a 90% confidence interval for the mean improvement in SAT scores for all Chicago city high school students is computed as (72.3, 91.4) points. The correct interpretation of this interval is

    (a) 90% of the students in the sample improved their scores by between 72.3 and 91.4 points.
    (b) 90% of the students in the population should improve their scores by between 72.3 and 91.4 points.
    (c) There is a 90% chance that the next randomly selected student will also have an improved score ranging from 72.3 to 91.4 points.
    (d) None of the above.

28. A fifth grade teacher gives homework every night in both mathematics and language arts. The time to complete the mathematics homework has a mean of 30 minutes and a standard deviation of 10 minutes. The time to complete the language arts assignment has a mean of 40 minutes and a standard deviation of 12 minutes. The time to complete the mathematics and the time to complete the language arts homework have a correlation $\rho = -0.3$. The mean time to complete the entire homework assignment

    (a) is less than 70 minutes since the negative correlation tells you that more time on one assignment will be associated with less time on the second assignment.
    (b) is 70 minutes.
    (c) is greater than 70 minutes since the measurements are correlated which raises the mean regardless of the sign of the correlation.
    (d) could be any time between 70 minutes minus 22 minutes to 70 minutes plus 22 minutes.

29. For a χ^2–test that is based on a table that consists of 2 rows and 3 columns, the value of the chi–square statistic is 7.422. The corresponding P–value is

    (a) less than 0.01.
    (b) between 0.01 and 0.05.
    (c) between 0.05 and 0.10.
    (d) greater than 0.10.

30. In the table below we examine the relationship between final grade and the reported hours per week each student said they studied for the course.

	A	B	C	D	F	Total
< 5	2	3	7	4	5	21
$5 - 10$	3	5	5	4	6	23
> 10	6	5	3	4	2	20
Total	11	13	15	12	13	64

The expected count of those who studied between 5 and 10 hours per week and earned a B for the course is

    (a) about 0.078.
    (b) about 4.672.
    (c) about 4.266.
    (d) about 8.265.

TABLES

You can remove the following 2 sheets of tables from the exam. No need to turn in these tables once they have been removed.

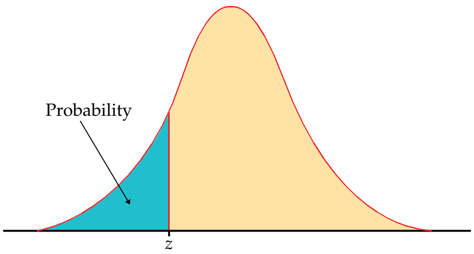

Table entry for z is the area under the standard normal curve to the left of z.

TABLE A

Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for *z* is the area under the standard normal curve to the left of *z*.

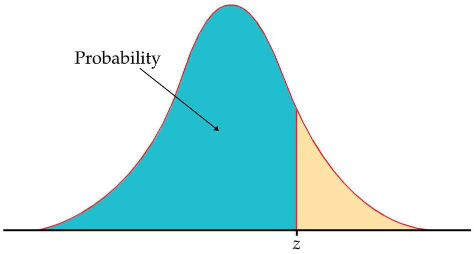

TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

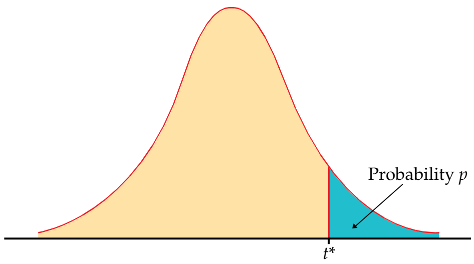

TABLE D

t distribution critical values

	Upper-tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

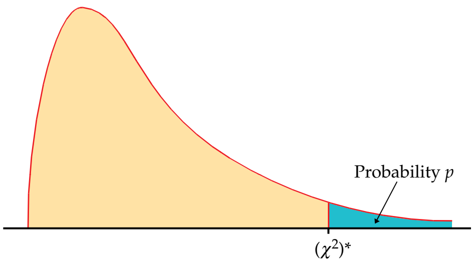

Table entry for p is the critical value $(\chi^2)^*$ with probability p lying to its right.

TABLE F

χ^2 distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2