

Final Test, May 4, 1:30pm–3:20pm

100 → **400**

Show your work. The test is out of 100 points and you have 110 minutes to finish.

- 8 → **32** 1. The following is part of an article from Newsweek, April 24, 2006. This question concerns the Women's Health Study, described in the second paragraph.

Take an Aspirin and ...

BY JULIE BURING, SC.D., AND
NANCY FERRARI

Aspirin is a wonder drug, plain and simple. At high doses, it quells inflammation; at medium doses, it provides effective pain relief; at low doses, it reduces the blood's ability to clot by inhibiting the action of tiny blood cells called platelets. It makes sense, then, that aspirin might help prevent clot-related cardiovascular events such as heart attack and stroke, even in healthy people. In 1988, the Physicians' Health Study showed exactly that. In healthy men, 325mg of aspirin taken every other day for five years reduced the risk of a

first heart attack by 44 percent. That was great news. For men.

It wasn't until March 2005 that the Women's Health Study addressed aspirin's benefits for women. Healthy women—who were at least 45 years old at the start of the study—who participated in the study took either 100mg of aspirin or a placebo every other day for 10 years. Surprisingly, the women taking aspirin experienced no reduction in heart-attack risk. However, aspirin takers were 17 percent less likely to have a stroke.

- 8** (a) (2 points) Is the study controlled? How do you know?

yes, it is controlled - women were given an aspirin (in the treatment group) or a placebo (in the control group) **2**

- 8** (b) (2 points) Is the study blind? How do you know?

yes, it is blind - as a placebo was given, women could not judge whether they were given the real aspirin treatment or not **2**

- 8** (c) (2 points) What is a placebo? Why is it used?

a placebo looks like the real treatment (i.e., aspirin here), but it has no active ingredients, **4** it is used to prevent that people respond to the idea **4** of treatment, rather than the treatment itself

- 8** (d) (2 points) The article does not say how the women were assigned to the aspirin and placebo groups. What is the best way to do this, and why?

Participants should be assigned randomly **4** to the two groups. This will reduce the effect of all possible confounding factors such as age, gender, etc. **4**

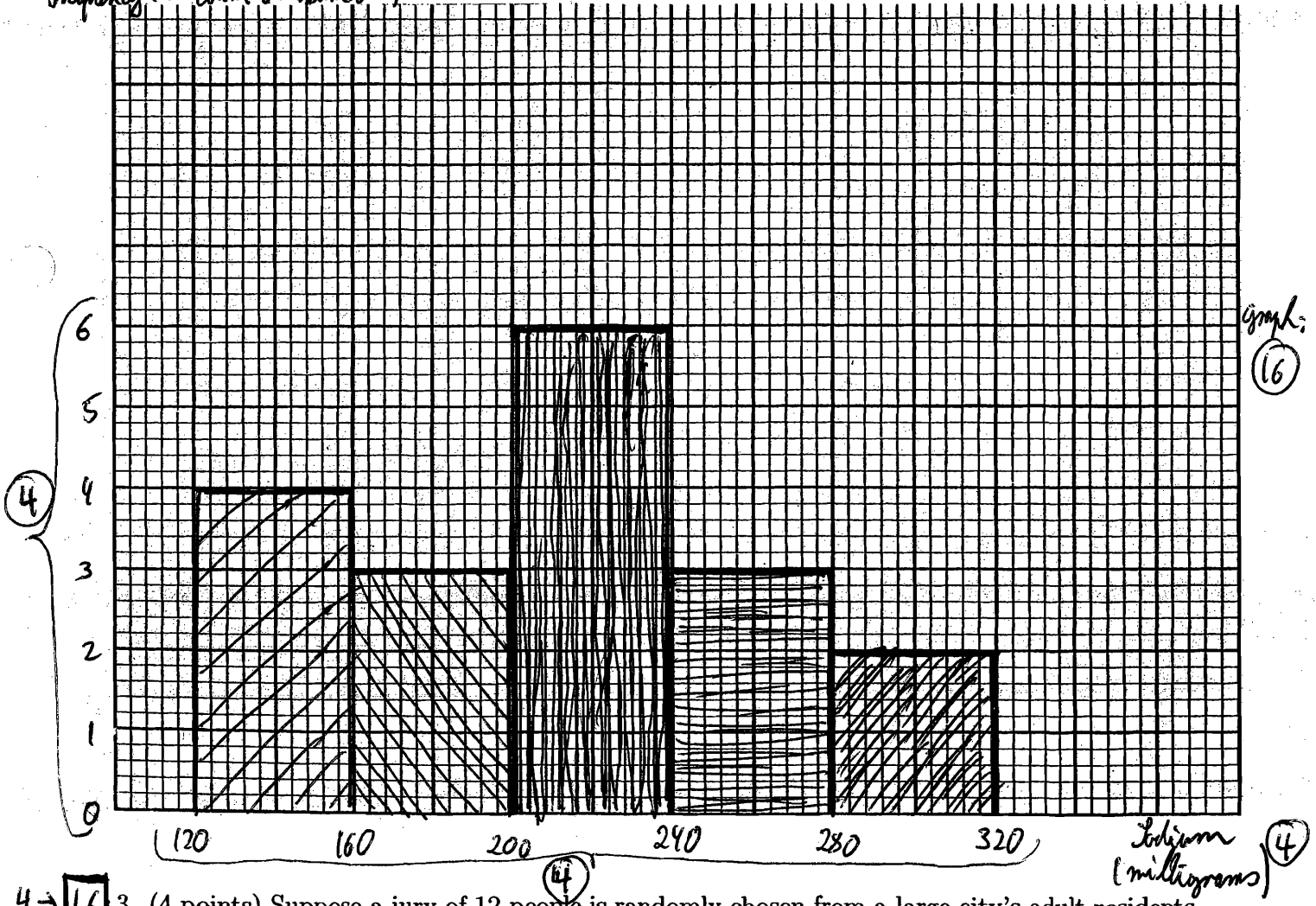
8 → 32

2. (8 points) The following table summarizes the number of milligrams of sodium per serving for 18 types of breakfast cereal. Class intervals include the left endpoint but not the right.

Sodium (milligrams)	Number of cereals
120 to 160	4
160 to 200	3
200 to 240	6
240 to 280	3
280 to 320	2

As each class is equally wide, we can just draw frequencies on the vertical axis (no need for the density scale - but, of course, you can use it if you really want to)

4 Draw a histogram for the data, being careful to label the axes correctly.



4 → 16

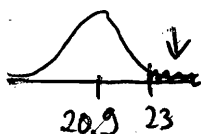
3. (4 points) Suppose a jury of 12 people is randomly chosen from a large city's adult residents, of whom 50% are female. What is the chance that at least one person on the jury is female?

$$\begin{aligned}
 \text{chance of at least one female} &= 1 - \text{chance of no female} = 1 - (0.5)^{12} \\
 &= 0.99976 = 99.976\%
 \end{aligned}$$

-2 for each calculation error

10 → 40 4. For 167 college students the average handspan size is 20.9 inches, with an SD of 1.9 inches.

20 (a) (5 points) Using the normal curve, approximately what percentage of the students have a handspan of more than 23 inches?



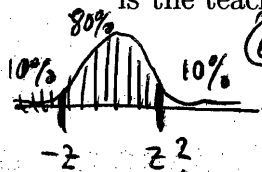
S.U.: $\frac{23-20.9}{1.9} = \frac{2.1}{1.9} = 1.10$

-2 for each calculation error

area between -1.10 and 1.10: 72.87% (6)

area above 1.10: $\frac{100\% - 72.87\%}{2} = \underline{\underline{13.57\%}}$ (6)

20 (b) (5 points) If 90 percent of the students have a smaller handspan than the teacher, what is the teacher's handspan?



area between -1.30 and 1.30: 80.64% (closest to 80%)

original units: $1.30 \cdot 1.9 + 20.9 = 2.47 + 20.9$

23.37 inches

14 → 56 5. For the 167 college students in question 4, the relationship between height and handspan size is summarized as follows:

x height: average = 68.0 inches SD = 4.0 inches r = 0.75

y handspan size: average = 20.9 inches SD = 1.9 inches

↑ about 95% of data within $20.9 \pm 2 \cdot 1.9 = 17.1$ to 24.7

16 (a) (4 points) Six scatter diagrams are printed on the back of the formula sheet. Which of the scatter diagrams is the correct one for these data? Circle the correct one below:

A (B) (16) C D E F

[see figure for explanation - not needed]

16 (b) (4 points) Using the summary statistics above, what is the regression estimate for handspan for a student who is 60 inches tall?

slope = $r \cdot \frac{SD_y}{SD_x} = 0.75 \cdot \frac{1.9}{4.0} = 0.36$ (5)

-2 for each calculation error
-1 if x,y not stated
-3 if x,y swapped

intercept = $avg_y - slope \cdot avg_x = 20.9 - 0.36 \cdot 68 = -3.58$ (5)

regression equation: $y = -3.58 + 0.36 \cdot x$ (6)

for height 60: $y = -3.58 + 0.36 \cdot 60 = \underline{\underline{18.02 inches}}$

12 (c) (3 points) Find the rms error for your answer in part (b).

-4 for each mistake

r.m.s. error = $\sqrt{1-r^2} \cdot SD_y = \sqrt{1-0.75^2} \cdot 1.9 = \underline{\underline{1.26}}$

12 (d) (3 points) What would the correlation coefficient be if we changed all the handspan measurements to centimeters? (There are 2.54 centimeters in an inch).

$r = 0.75$ (it won't change)

(12)

- 4 → **16** (4 points) I have 20 batteries, of which 15 work and the other 5 don't. I choose 4 batteries at random from all 20 to put in my camera. What is the chance that all 4 batteries work?

1st works $\frac{15}{20}$ (4) (1) 2nd works, given 1st works $\frac{14}{19}$ (3) (1) 3rd works, given 1st & 2nd work $\frac{13}{18}$ (2) (1) 4th works, given 1st, 2nd & 3rd work $\frac{12}{17}$ (3) (1)

-2 for each calculation error
= 0.2817 = 28.17%

- 8 → **32** (8 points) In 2002, 17.5% of US tax forms reported incomes under \$25,000. If the IRS takes a random sample of 500 tax forms from 2002, what is the chance that between 17% and 18% of the sampled forms reported incomes of under \$25,000?



base: $\frac{175 \times 1 + 825 \times 0}{\# \text{ draws: } 500}$ (3) (2)

1: under \$25K
0: above \$25K

base avg = 0.175 (4)
base SD = $\sqrt{0.175 \cdot 0.825} = 0.38$ (4)
EV % = 17.5% (3)
SE_{sam} = $\sqrt{500} \cdot 0.38 = 8.5$ (4)
SE_% = $\frac{8.5}{500} \cdot 100\% = 1.7\%$ (4)

S.a.: $\frac{17\% - 17.5\%}{1.7\%} = -0.29$ (4)
 $\frac{18\% - 17.5\%}{1.7\%} = 0.29$ (4)
area between -0.30 and 0.30:
23.58% (4)

- 9 → **36** (8 points) The 2002 General Social Survey asked "What do you think is the ideal number of children for a family to have?" For the 497 females who responded, the average was 3.02 with a standard deviation of 1.81. Suppose these 497 women were like a simple random sample of adult American females.

- 28** (a) (7 points) Find a 95% confidence interval for the average response of all adult American females.

avg = 3.02
SD = 1.81
SE_{sam} = $\sqrt{497} \cdot 1.81 = 40.35$ (4)
SE_{avg} = $\frac{40.35}{497} = 0.08$ (4)

-2 for each calculation error

95% CI: $3.02 \pm 2 \cdot 0.08 = \underline{2.86}$ to $\underline{3.18}$
(4) (2) (4) (2) (4) (2)

- 8** (b) (2 points) Is your confidence interval valid even though the answers don't follow the normal curve? Explain briefly.

(4) yes, it is still valid; the probability histogram of the average will follow
(6) the normal curve even if the data do not (2)

11 → 44 9. (20 points) The following table comes from a simple random sample of high school seniors from a large city, each of whom was asked whether they had ever smoked cigarettes and whether they had ever drunk alcohol.

		obs. count		
		No	Yes	
Cigarettes	No	281	500	781
	Yes	46	1449	1495
		327	1949	2276

exp. count

①: 112	②: 669	781
③: 215	④: 1280	1495
327	1949	2276

-2 for each calculation error
-30 for incorrect test

-4 if null, alt swapped
 $4 \times ② = ⑧$

Test to see whether there is an association between smoking and drinking alcohol. You should clearly state the null and the alternative hypotheses, find a test statistic and an approximate P-value, and state your conclusions in everyday language.

χ^2 -test for independence:

1) null: cigarette and alcohol consumption are independent, i.e., boxes are identical ①
alternative: cigarette and alcohol consumption are not independent, i.e., boxes are different ①

2) expected: ① = $\frac{781 \cdot 327}{2276} = 112$ ② = $781 - 112 = 669$

③ = $327 - 112 = 215$ ④ = $1949 - 669 = 1280$

$$\chi^2 = \text{sum of } \frac{(\text{obs} - \text{exp})^2}{\text{exp}} = \frac{(281 - 112)^2}{112} + \frac{(500 - 669)^2}{669} + \frac{(46 - 215)^2}{215} + \frac{(1449 - 1280)^2}{1280}$$

$$df = (2-1)(2-1) = 1 \text{ ④} = 452.9 \text{ ⑥}$$

3) $\chi^2 = 452.9$ is to the right of 6.64 ②

→ P-value is to the right of 1%, i.e., P-value < 1% ④

4) reject the null (P-value < 5%) ④
• result is highly stat. sig. (P-value < 1%) ④
• cigarette and alcohol consumption are not independent ④

11 → 44 10. (20 points) In the aspirin study described in question 1, there were 19,934 women in the aspirin group and 19,942 in the placebo group. There were 477 major cardiovascular events in the aspirin group and 522 in the placebo group. Assuming the women were assigned to the aspirin and placebo groups appropriately, perform a statistical significance test to determine whether or not aspirin prevents major cardiovascular events for women like these. You should clearly state the null and the alternative hypotheses, find a test statistic and an approximate P-value, and state your conclusions in everyday language.

2-sample z-test:

A: aspirin group

C: control group (placebo)

-2 for each calculation error

1) null: A and C have cardiovascular events at the same rate, i.e., $\text{box } A\% - \text{box } C\% = 0\%$ ①

alternative: A have cardiovascular events at a lower rate, i.e., $\text{box } A\% - \text{box } C\% < 0\%$ ①

$$SE_{\text{diff}} = \sqrt{(0.11\%)^2 + (0.11\%)^2} = 0.16\% \text{ ④}$$

$$z = \frac{2.39\% - 2.62\%}{0.16\%} = -1.44 \text{ ④}$$

3) area between -1.45 and 1.45: 85.29% ③

$$P\text{-value} = \text{area below } -1.45 = \frac{100\% - 85.29\%}{2} = 7.4\% \text{ ③}$$

2)

$$\text{sample size } A = 19934$$

$$\text{sample } A\% = \frac{477}{19934} = 2.39\% \text{ ②}$$

$$SD_A = \sqrt{0.0239 \cdot 0.9761} = 0.15 \text{ ②}$$

$$SE_{\text{sum } A} = \sqrt{19934} \cdot 0.15 = 21.2 \text{ ②}$$

$$SE\% A = \frac{21.2}{19934} \cdot 100\% = 0.11\% \text{ ②}$$

$$\text{sample size } C = 19942$$

$$\text{sample } C\% = \frac{522}{19942} = 2.62\% \text{ ②}$$

$$SD_C = \sqrt{0.0262 \cdot 0.9738} = 0.16 \text{ ②}$$

$$SE_{\text{sum } C} = \sqrt{19942} \cdot 0.16 = 22.6 \text{ ②}$$

$$SE\% C = \frac{22.6}{19942} \cdot 100\% = 0.11\% \text{ ②}$$

4) do not reject the null (P-value > 5%) ④
• A and C have cardiovascular events at the same rate ④

13 → 52 11. The long-held standard for normal body temperature is 98.6 degrees. A group of researchers believe that normal body temperature is actually lower than 98.6 degrees. To test this, they take the temperatures of 10 healthy people. They find the following:

98.2 97.8 99.0 98.6 97.8 98.4 99.7 98.2 97.4 97.6

Suppose this is a simple random sample of all healthy people and that the temperatures of healthy people follow the normal curve.

-2 for each calculation error
-30 for incorrect test
-4 if null alt swapped

8 (a) (2 points) State the appropriate null and alternative hypotheses.

t-test:
• sample size < 30 ✓
• SD for pop unknown ✓
• data follows normal curve

H_0 : body temperature is as expected, i.e., $\mu_{\text{avg}} = 98.6$ (1)
alternative: body temperature is less than expected, i.e., $\mu_{\text{avg}} < 98.6$ (1)

12 (b) (3 points) Find a test statistic.

2) observed (avg) = $\frac{98.2 + 97.8 + \dots + 97.4 + 97.6}{10} = 98.3$ (2)

expected (avg) = 98.6

SD = $\sqrt{\frac{(98.2 - 98.3)^2 + (97.8 - 98.3)^2 + \dots + (97.4 - 98.3)^2 + (97.6 - 98.3)^2}{10}} = 0.66$ (2)

SD* = $\sqrt{\frac{10}{9}} \cdot 0.66 = 0.70$ (2)

SE_{sum} = $\sqrt{10} \cdot 0.70 = 2.2$ (2)

SE_{avg} = $\frac{2.2}{10} = 0.22$ (2) | $t = \frac{98.3 - 98.6}{0.22} = -1.36$ (2)

8 (c) (2 points) How big is the P-value?

3) df = 10 - 1 = 9 (2)

$t = +1.36$ falls between 0.70 and 1.38 (2)

→ P-value falls between 25% and 10% (3)

8 (d) (2 points) Do you reject the null hypothesis? Why/why not?

4) • do not reject the null (P-value > 5%) (4)

8 (e) (2 points) Based on the data, are the researchers correct? Explain briefly.

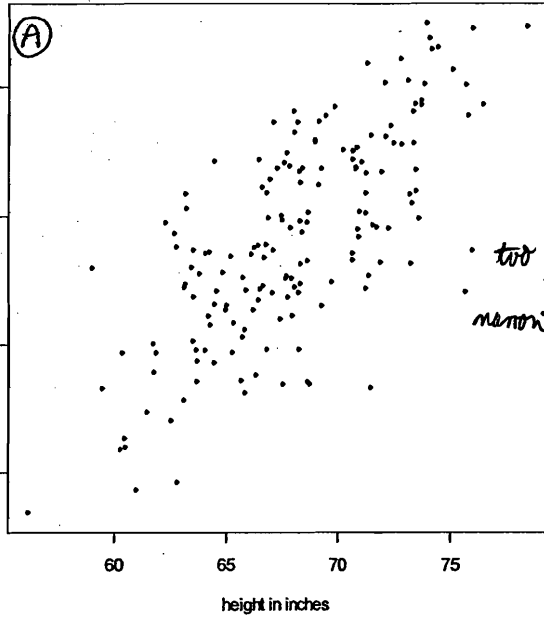
• (no) they are not correct; body temperature is as expected (4)
(the observed temperature is only due to chance error)

8 (f) (2 points) If it was believed that the temperatures of healthy people did not follow the normal curve, would your hypothesis test still be valid? Explain briefly.

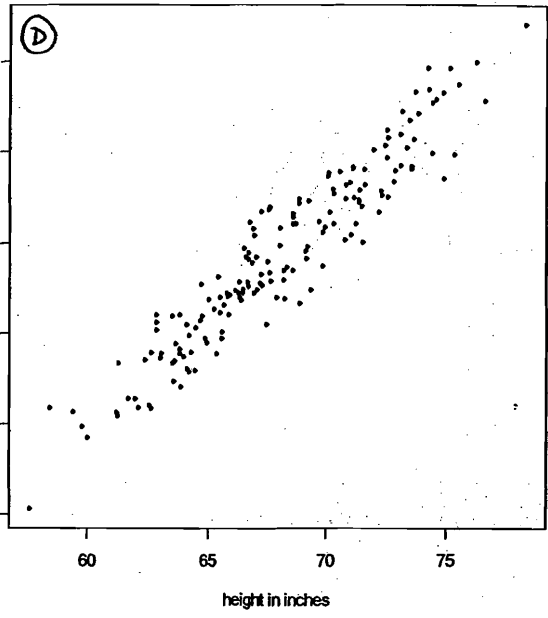
(no) it would become invalid: for the t-test, the data must follow the normal curve (4)

Exclude impossible plots

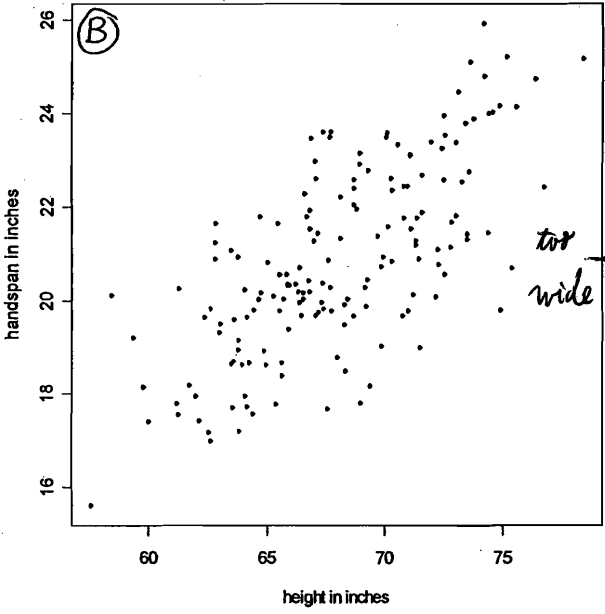
too narrow



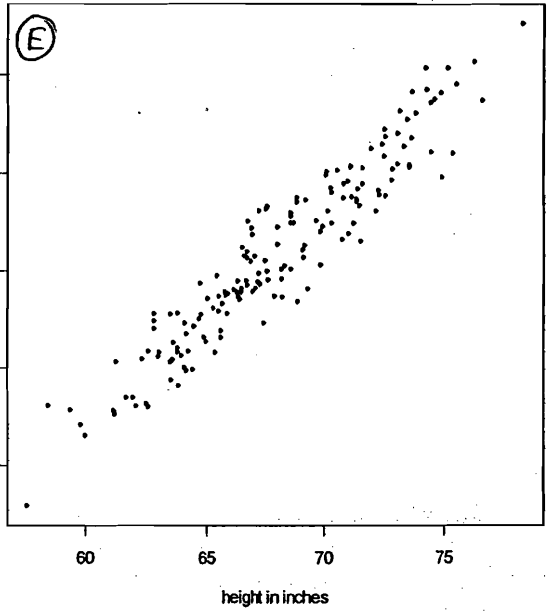
too narrow



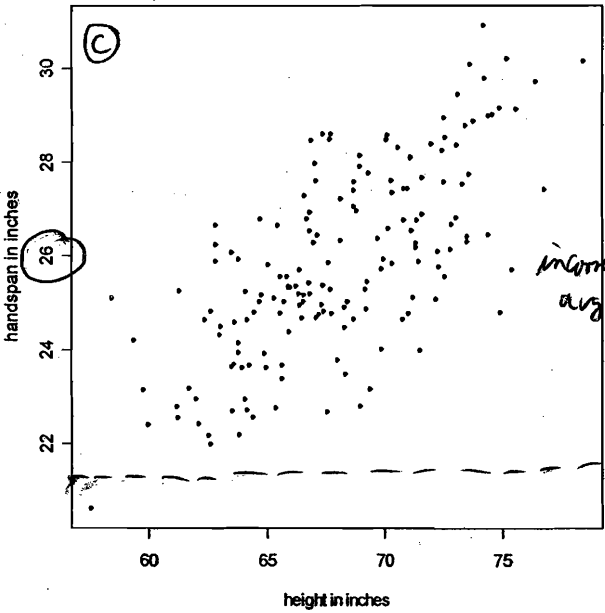
This is it!



too wide



incorrect axis



incorrect axis

