Stat 1040, Spring 2005 **Name:** ____________________

Final Test, May 3, 1:30pm–3:20pm

100 → $\boxed{400}$

Show your work. The test is out of 100 points and you have 110 minutes to finish.

Questions 1 to 5 below are based on survey results related to the Terri Schiavo case, as published in *Time*, April 4, 2005 (pp. 22–30). Terri Schiavo suffered severe brain damage during cardiac arrest in 1990. Over the next 15 years, Terri was kept alive through a feeding tube. For 10 years, Terri's husband, Michael Schiavo and Terri's parents, the Schindlers, battled in court over the issue of whether or not to remove Terri's feeding tube. After a final court order in March 2005, Terri's feeding tube was removed and Terri was allowed to die in early April 2005.

10 → $\boxed{40}$ 1. A simple random sample of 1010 Americans was polled by *Time* on the following question.

Question A: *"Do you agree with the decision to remove Schiavo's feeding tube?"*

Time reports that of the 1010 people,

595 (about 59%) **agree**

349 (about 35%) **disagree**

66 (about 7%) **don't know.**

$\boxed{12}$ (a) (3 points) Assume that Terri's parents, who strongly opposed the removal of Terri's feeding tube, initiated an alternative poll with the following question:

Question B: *"Do you agree with the decision to remove Terri's feeding tube, thus letting her starve to a slow and painful death?"*

Other things being equal, the percentage of people that would have answered agree to question (B) would have been (a) higher than 59%, (b) about 59%, or (c) much less than 59%. Circle your answer and explain clearly.

(8)

The wording of Question B is more personal (Terri instead of Schiavo) and includes a second part with a very negative wording ("starve to a slow and painful death") hardly anyone would agree with. The percentage of people that would agree would be much less than 59%, apparently supporting Terri's parents not to remove her feeding tube. (4)

$\boxed{16}$ (b) (4 points) *Time* reports that *"the margin of error for the percentage who agree is ±3 percentage points."* Is this (a) our usual standard error, (b) about twice our usual standard error, or (c) about one half of our usual standard error. Circle your answer and show the necessary calculations. (4)

via bootstrap:

box SD $= \sqrt{0.59 \cdot 0.41} = 0.49$ (4)

$SE_{sum} = \sqrt{1010} \cdot 0.49 = 15.57$ (4)

$SE\,\% = \frac{15.57}{1010} \cdot 100\% = 1.54\%$ (4)

so: $\frac{3\%}{1.54\%} = 1.95 \approx 2$, i.e., twice our usual standard error

$\boxed{12}$ (c) (3 points) *Time* further presents outcomes for different subgroups (such as for church goers/non church goers and for different political affiliations) and states that *"the margin of error is higher for subgroups"*. Assume that we are looking at a subgroup that consists of about 253 people. For these 253 people, the standard error for the percentage who **agree** would be

i. about four times as big

Note: $\frac{253}{1010} = \frac{1}{4}$

12 ii. about twice as big

iii. about $\sqrt{2}$ as big

so: SE is about $\frac{1}{\sqrt{1/4}}$ = 2 times as big as the previous SE

as the standard error for the sample of 1010 people. Just circle your answer – no calculation is necessary.

9 → $\boxed{36}$ 2. A newspaper reporter wants to do some follow–up interviews with some of the participants in the survey and randomly selects 3 *different* people from the original 1010 participants. Recall that 595 answered **agree**, 349 answered **disagree**, and 66 answered **don't know** to the Question (A).

−2 for each calculation error

−2 if no final % given

$\boxed{12}$ (a) (3 points) The chance that the 3 randomly selected participants for the follow–up interview all answered **agree** to question (A) is about 20.4 %.

1st agrees: $\frac{595}{1010}$ (2)

2nd agrees, given first agrees: $\frac{594}{1009}$ (3)

3rd agrees, given 1st & 2nd agree: $\frac{593}{1008}$ (3)

all 3 agree: $\frac{595}{1010}$ (2) $\cdot \frac{594}{1009}$ (2) $\cdot \frac{593}{1008} = 0.204 = 20.4\%$

$\boxed{12}$ (b) (3 points) The chance that the 3 randomly selected participants for the follow–up interview all answered **disagree** to question (A) is about 4.1 %.

1st disagrees: $\frac{349}{1010}$ (2)

2nd disagrees, given 1st disagrees: $\frac{348}{1009}$ (3)

3rd disagrees, given 1st & 2nd disagree: $\frac{347}{1008}$ (3)

all 3 disagree: $\frac{349}{1010}$ (2) $\cdot \frac{348}{1009}$ (2) $\cdot \frac{347}{1008} = 0.041 = 4.1\%$

$\boxed{12}$ (c) (3 points) The chance that *at least 1* of the 3 randomly selected participants for the follow–up interview answered **agree** to question (A) is about 93.1 %.

1st does not agree: $\frac{349+66}{1010} = \frac{415}{1010}$ (4)

2nd does not agree, given 1st does not agree: $\frac{414}{1009}$ (1)

3rd does not agree, given 1st & 2nd do not agree: $\frac{413}{1008}$ (1)

all 3 do not agree: $\frac{415}{1010}$ (1) $\cdot \frac{414}{1009}$ (1) $\cdot \frac{413}{1008} = 0.069 = 6.9\%$

at least 1 agrees: 100% (4) $- 6.9\% = 93.1\%$

10 → $\boxed{40}$ 3. Use the fact that about 59% of the 1010 Americans in the *Time* sample answered **agree** to Question (A).

− 2 for each calculation error

−8 if no final %'s given

$\boxed{28}$ (a) (7 points) Find an approximate 99% confidence interval for the percentage of **all** Americans that would have answered **agree** to Question (A).

via bootstrap:

$SD = \sqrt{0.59 \cdot 0.41} = 0.49$ (4)

$SE_{sum} = \sqrt{1010} \cdot 0.49 = 15.57$ (4)

$SE_{\%} = \frac{15.57}{1010} \cdot 100\% = 1.54\%$ (4)

or note that this is just the same $SE_{\%}$ as in 1(b) and use this number (1.54%) directly!

(4) (4) (4) (4)

99% CI: $59\% \pm 2.60 \cdot 1.54\%$

$= 59\% \pm 4\%$

$= 55\%$ to 63%

12 (b) (3 points) Is your confidence interval still valid if you find out that the 59% did not come from a simple random sample but from a Web page that belonged to Terri's husband (who strongly supported the removal of Terris' feeding tube)? Explain briefly.

No, not valid: We need a SRS to correctly calculate a CI. (4)

(8)

10 → **40** 4. (10 points) *Time* reports that 53% of Republicans answered **agree** to Question (A). The "religious right" in the Republican Party does not believe the percentage should be so high. To obtain a better idea about whether the majority of their members really **agree**, the Republican Party decides to do a simple random sample of Republicans. Suppose that the true (population) percentage of Republicans who **agree** is indeed 53%. What is the chance that in a simple random sample of 2600 Republicans, less than 1300 would answer **agree**? Indicate the box model, calculate the required EV's and SE's, and argue using the normal curve.

−2 for each calculation error

box: | 53 × $\boxed{1}$ 47 × $\boxed{0}$ | (4) 1: agree, 0: disagree

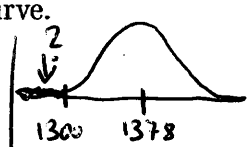
#draws: 2600 (2)

box avg = 0,53 (4)

box SD = $\sqrt{0.53 \cdot 0.47}$ = 0.50 (4)

EV_{sum} = 2600 · 0,53 = 1378 (6)

SE_{sum} = $\sqrt{2600}$ · 0,50 = 25,5 (6)



$z = \frac{1300 - 1378}{25.5} = -3.06$ (6)

area between −3.05 and 3.05: 99,771% (4)

area below −3,05: $\frac{100\% - 99.771\%}{2}$ = $\boxed{0.115\%}$ (4)

12 → **48** 5. (12 points) Some political experts believe that Democrats **agree** with question (A) at a higher rate than Independents. In fact, *Time* reports that 65% of Democrats **agree** whereas only 59% of Independents **agree**. Suppose that 450 Democrats and 180 Independents were initially included in the survey. Perform a statistical test to determine whether the political experts' claim is correct. You must state a null and alternative hypothesis, compute a test statistic and P–value, and clearly state your conclusions about whether the political experts are correct or not.

D: Democrats

I: Independents

−2 for each calculation error

−36 for incorrect test

−4 if null/alt. assumed

2-sample z-test: (3)

1) null: D and I agree at the same rate, i.e., $box_{D\%} - box_{I\%} = 0\%$ (1)

alternative: D agree at a higher rate, i.e., $box_{D\%} - box_{I\%} > 0\%$ (1) (3)

2) **D**

sample size D = 450

sample D% = 65%

$SD_D = \sqrt{0.65 \cdot 0.35}$ = 0.48 (3)

$SE_{sum_D} = \sqrt{450}$ · 0.48 = 10,18 (3)

$SE_{\%_D} = \frac{10.18}{450} \cdot 100\%$ = 2,26% (3)

I

sample size I = 180

sample I% = 59%

$SD_I = \sqrt{0.59 \cdot 0.41}$ = 0.49 (3)

$SE_{sum_I} = \sqrt{180}$ · 0.49 = 6.57 (3)

$SE_{\%_I} = \frac{6.57}{180} \cdot 100\%$ = 3.65% (3)

$SE_{diff\%} = \sqrt{(2.26\%)^2 + (3.65\%)^2}$ = 4.29% (4)

$z = \frac{65\% - 59\%}{4.29\%} = 1.40$ (3)

3) area between −1.40 and 1.40: 83.85% (3)

⇝ P-value: $\frac{100\% - 83.85\%}{2}$ = 8.08% (3)

4) • do <u>not</u> reject the null (P-value > 5%) (3)

• D and I agree at the same rate (3)

• experts are <u>incorrect</u>! (3)

11 → $\boxed{44}$ 6. In a recent news article, a researcher claims that being overweight in middle age causes dementia later in life, although the mechanism is unknown. The dementia rate in overweight people was about 8%, while in normal-weight "controls" it was only 7%.

$\boxed{8}$ (a) (2 points) The difference between dementia rates for overweight and normal-weight people was "highly statistically significant". What does this tell you about the P-value?

The p-value is less than 1% (p-value < 1%). 8

$\boxed{12}$ (b) (3 points) Consider the following statement:

The difference was "highly statistically significant" so if overweight people don't lose weight, they are extremely likely to get dementia.

Is this what statistical significance tells us? Explain.

4 No! Association is not the same as causation. 4

There might be confounding factors. 4

$\boxed{8}$ (c) (2 points) Is this an observational study 6 or a controlled experiment? Explain.

2 No intervention took place — they just observed participants.

$\boxed{16}$ (d) (4 points) Suggest a confounding factor and clearly explain why this factor might be responsible for the association between weight and dementia.

Possible confounding factors (just one needed):

- overall health: overweight and dementia might be linked to the same medical problem — explanation: 8
- gender: overweight and dementia might differ among men and women
- age: it may take some time to develop dementia (an 80 year old may have had, but a 60 year old may not have had)
- length and amount of being overweight: does it matter if someone is 10 pounds overweight for 50 years, or 50 pounds overweight for 10 years?

8

−2 if x, y not stated
−4 if x, y swapped

14 → 56 7. Leonardo da Vinci (1452–1519) theorized that if you put your arms out to the side and measured from the fingertip of one hand to the fingertip of the other, this 'wingspan' distance would approximately equal your height. A group of fourth–grade students measured their height and wingspan and found

−2 for each calculation error

average height (x) = 49.5 inches with an SD of 1.8 inches average wingspan (y) = 48.9 inches with an SD of 2.1 inches

The scatter diagram was football-shaped and the correlation coefficient was 0.8.

20 (a) (5 points) Find the equation of the regression line for predicting wingspan from height.

x = height
y = wingspan

$$\text{slope} = r \cdot \frac{SD_y}{SD_x} = 0.8 \cdot \frac{2.1}{1.8} = 0.933$$

(3) (2) (2)

$$\text{intercept} = \text{avg}_y - \text{slope} \cdot \text{avg}_x = 48.9 - 0.933 \cdot 49.5 = 2.72$$

(2) (2) (2) (1) (2)

regression equation: $\boxed{y = 2.72 + 0.933 \cdot x}$ or $\boxed{\text{wingspan} = 2.72 + 0.933 \cdot \text{height}}$ (4)

8 (b) (2 points) Does the intercept make sense? Explain.

(4) Yes, it does. The intercept represents the predicted wingspan for a child that is 0 inches tall. Note that 0 inches is $\frac{49.5}{1.8} = 27.5$ SD below the average height – so this would be extreme extrapolation. Whatever the intercept is, it would make sense here. (4)

12 (c) (3 points) Predict the wingspan of a fourth–grader who is 52 inches tall.

for height = 52,

$$\text{wingspan} = 2.72 + 0.933 \cdot 52 = \boxed{51.24 \text{ inches}}$$ (12)

8 (d) (2 points) Find the rms error for your answer in part (b).

$$\text{r.m.s. error} = \sqrt{1 - r^2} \cdot SD_y = \sqrt{1 - 0.8^2} \cdot 2.1 = \sqrt{1 - 0.64} \cdot 2.1$$

(2) (2) (2) (2)

$$= \sqrt{0.36} \cdot 2.1 = 0.6 \cdot 2.1 = \boxed{1.26 \text{ inches}}$$

8 (e) (2 points) One of these fourth–graders is 52 inches tall. Is it likely that he or she would have a wingspan of 45 inches? Explain clearly using your answers to (c) and (d).

predicted wingspan for 52 inches: 51.24 inches (from (c))

$$\text{S.U.}: \frac{45 - 51.24}{1.26} = \frac{-6.24}{1.26} = -4.95$$ (3)

(3) No, very unlikely. A wingspan of 45 inches is more than 3 rms errors below the predicted wingspan of 51.24 inches. Almost 0% of children who are 52 inches tall would have such a short (or even shorter) wingspan. (2)

12 → $\boxed{48}$ 8. (12 points) The following table comes from a simple random sample of university students, each of whom was asked whether they agree or disagree with the statement "Cell phone use while driving should be prohibited".

−2 for each calculation error

−36 for incorrect test

−4 if null, alt swapped

	Male	Female	
Agree	13	16	29
Disagree	240	206	446
	253	222	475

expected:

15.4	13.6
237.6	208.4

4 × 4 = 16

Test to see whether male and female university students differ in their responses. You should clearly state the null and the alternative hypothesis, find a test statistic and an approximate P-value, and state your conclusions in everyday language.

χ^2-test for independence:

1) null: gender and agreement are independent (3), i.e., boxes are identical (1)

alternative: gender and agreement are not independent (3), i.e., at least one box is different (1)

2) expected: $\frac{29 \cdot 253}{475} = 15.4$ etc. (see table "expected" above)

$\chi^2 = \text{sum of } \frac{(\text{obs}-\text{exp})^2}{\text{exp}} = \frac{(13-15.4)^2}{15.4} + \frac{(16-13.6)^2}{13.6} + \frac{(240-237.6)^2}{237.6} + \frac{(206-208.4)^2}{208.4} = 0.85$ (6)

df $= (2-1)\cdot(2-1) = 1$ (2)

3) $\chi^2 = 0.85$ between 0.46 and 1.07
$\rightsquigarrow$ P-value between 50% and 30% (6)

4) • do not reject the null (P-value > 5%) (5)
• gender and agreement are independent (5)

12 → $\boxed{48}$ 9. (12 points) An English exam is taken by 2000 students. The exam scores are known to follow the normal curve. The teacher says that the average of all 2000 test scores is 75, but one of the students thinks the average is actually lower. She takes a simple random sample of 9 students and finds they got the following scores:

63, 53, 84, 82, 35, 50, 68, 73, 92

−2 for each calculation error

−36 for incorrect test

−4 if null, alt swapped

Test to determine whether the average really is 75, against the alternative that the student is correct. You should clearly state the null and the alternative hypothesis, find a test statistic and an approximate P-value, and state your conclusions in everyday language.

t-test: • sample size < 30 ✓
• SD for box unknown ✓
• data follow normal curve ✓

1) null: avg of all test scores is as indicated, (3) i.e., box avg = 75 (1)

alternative: avg of all test scores is less than indicated, (3) i.e., box avg < 75 (1)

2) sample avg $= \frac{63+53+84+82+35+50+68+73+92}{9} = 66.67$ (4)

sample SD $= \sqrt{\frac{(63-66.67)^2 + \dots + (92-66.67)^2}{9}} = \sqrt{\frac{2700}{9}} = 17.32$ (4)

$SD^+ = \sqrt{\frac{9}{9-1}} \cdot 17.32 = 18.37$ (4)

$SE_{sum} = \sqrt{9} \cdot 18.37 = 55.11$ (4)

$SE_{avg} = \frac{55.11}{9} = 6.12$ (4)

$t = \frac{66.67-75}{6.12} = \frac{-8.33}{6.12} = -1.36$ (4)

df $= 9-1 = 8$ (4)

3) look up $t = 1.36$ (and not -1.36):
$t = 1.36$ between 0.71 and 1.40
$\rightsquigarrow$ P-value between 25% and 10% (4)

4) • do not reject the null (P-value > 5%) (4)
• avg of all test scores could be as indicated (i.e., avg could be 75) (4)