Name: ______________________

Stat 1040, Fall 2003

Final Test, Monday December 8, 7:30 – 9:20 am

100 → $\boxed{400}$

Show your work. The test is worth 100 points and you have 110 minutes. −2 for each calculation error

15 → $\boxed{60}$ 1. A tire manufacturer claims that the average life of a certain grade of tire is 25,000 (or more) miles under normal driving conditions. A random sample of 15 tires is tested. The mean and standard deviation are 22,500 and 5,000 miles, respectively. Assume that the lives of the tires are approximately normally distributed. Can we conclude from these data that the manufacture's product is as good as claimed?

−27 for incorrect test
−3 if null, alt swapped

$\boxed{36}$ a. (9 points) State the null and alternative hypothesis, perform the appropriate statistical test, and clearly state your conclusions.

t-test: • sample size < 30 ✓
• SD for box unknown ✓
• data follows normal curve ✓

1) null: tires have the claimed life span, i.e., avg = 25,000 (4)
alternative: tires have shorter life span, i.e., avg < 25,000 (4)

2) $SD^+ = \sqrt{\frac{15}{15-1}} \cdot 5{,}000 = 1{,}035 \cdot 5{,}000 = 5175$ (3)

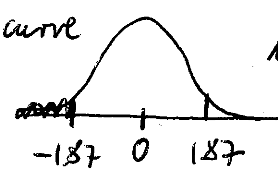
$SE_{sum} = \sqrt{15} \cdot 5175 = 3.87 \cdot 5175 = 20027$ (3)

$SE_{avg} = \frac{20{,}027}{15} = 1{,}335$ (3)

$t = \frac{22{,}500 - 25{,}000}{1{,}335} = \frac{-2{,}500}{1{,}335} = -1.87$ (3)

$df = 15 - 1 = 14$ (3)

3) t-curve



use 1.87 in t-table

t = 1.87 between 1.76 and 2.14
↓ 5% ↓ 2.5%

⇝ P-value between 2.5% and 5% (4)

4) • reject the null (P-value < 5%) (3)
• result is statistically significant (3)
• tires have shorter life span than 25,000 miles (3)

$\boxed{12}$ b. (3 points) Suppose that tire life is not normally distributed, but has a long left tail. Can we rely on the results from part (a) above? Justify your answer.

No! (10) The t-test only produces valid results if the data follows the normal curve. (2)

$\boxed{12}$ c. (3 points) If you rejected the null hypothesis in part (a) above, have you shown that manufacture's claims cannot possibly be true?

No! (10) The P-value indicates that the chance for such an observed (or even more extreme) outcome is between 2.5% to 5%. This is not a very big chance, but this is far from being impossible. (2)

A: natural sample size: 300 sample $\%_A = 45/300 = 15\%$
$SD_A = \sqrt{.15 \cdot .85} = .357$ 2
$SE_{sum A} = \sqrt{300} \cdot .357 = 6.18$ 2
$SE\%_A = \frac{6.18}{300} \cdot 100\% = 2.06\%$ 2

B: synthetic sample size: 350 sample $\%_B = 70/350 = 20\%$
$SD_B = \sqrt{.2 \cdot .8} = .4$ 2
$SE_{sum B} = \sqrt{350} \cdot .4 = 7.48$ 2
$SE\%_B = \frac{7.48}{350} \cdot 100\% = 2.14\%$ 2

10 → $\boxed{40}$ 2. (10 points) A firm that makes carpet is seeking a material that can withstand temperatures of up to 250° F. Two materials, one a natural material, and the other a synthetic (and cheaper) material, are equally satisfactory in all respects except, possibly, heat tolerance. A simple random sample of 300 specimens of the natural material is drawn, of which 45 fail at temperatures below 250° F. A simple random of sample of 350 specimens of the synthetic material is drawn of which 70 fail at temperatures below 250° F. Can we conclude from these data that the synthetic material is less heat tolerant? State the null and alternative hypothesis, perform the appropriate statistical test, and clearly state your conclusions.   1: failed   0: did not fail

−30 for incorrect test
−3 if null, alt swapped

2-sample z-test:

1) null: natural & synthetic material have the same heat tolerance, i.e., $lose_{A\%} - lose_{B\%} = 0\%$ 4
alternative: synthetic material is less heat tolerant, i.e., $lose_{A\%} - lose_{B\%} < 0\%$ 4

2) $SE_{diff\%} = \sqrt{2.06\%^2 + 2.14\%^2} = 2.97\%$ 4

$z = \frac{(15\% - 20\%) - 0\%}{2.97\%} = \frac{-5\%}{2.97\%} = -1.68$ 4

3) area between −1.70 and 1.70: 91.09%
⇝ P-value $= \frac{100\% - 91.09\%}{2} = 4.46\%$ 4

4) • reject the null (P-value < 5%) 3
• result is statistically significant 2
• synthetic material is less heat tolerant 3

9 → $\boxed{36}$ 3. "HEIGHT MATTERS for career success", said Timothy Judge, a University of Florida management professor whose research will appear in the spring issue of the Journal of Applied Psychology. Judge and Daniel Cable, a business professor at the University of North Carolina at Chapel-Hill, analyzed results from four large-scale studies in the U.S. and Britain that followed thousands of participants from childhood to adulthood, examining details of their work and personal lives. The study controlled for gender, weight and age, and found that each inch in height added about $789 a year in pay.

9

$\boxed{12}$ a.(3 points) Was the described study an **observational study** or a **controlled experiment?** Circle one and explain briefly.

No intervention took place – the participants were just followed. 3

9

$\boxed{12}$ b. (3 points) Was the described study **cross-sectional** or **longitudinal?** Circle one and explain briefly.

The participants were followed from childhood to adulthood, i.e., for many years. 3

$\boxed{12}$ c. (3 points) What does it mean "The study controlled for gender, weight and age."? Explain why this is important.

8

These may be confounding factors that could have an effect on the salary earned. Men often make more money than women but also are taller. Adults obviously make more money than children and are taller. 4

13→ $\boxed{52}$

4. A telephone answering service, at the end of each call, completes a report in which the length of the call is recorded. A simple random sample of 150 reports yields a mean length per call of 1.2 minutes with a standard deviation of 0.4 minutes.

$\boxed{16}$ a. (4 points) Construct a 95% confidence interval for the average length of all the calls handled by the answering service.

$SD = 0.4$

$SE_{sum} = \sqrt{150} \cdot 0.4 = 4.9$ (5)

$SE_{avg} = \frac{4.9}{150} = 0.0327 \approx 0.03$ (5)

95% CI : 1.2 min $\pm 2 \cdot 0.03$ min (4) $=$ 1.14 min to 1.26 min (2)

$\boxed{12}$ b. (3 points) Because some calls are quite lengthy, call length does not follow the normal curve; it has a long right tail. Does this mean that your confidence interval calculated above is incorrect? Briefly explain.

(10) No! We are looking at the average and this will follow the normal curve even if the original data does not. (2)

$\boxed{8}$ c. (2 points) True or False (please circle your choice):
95% of the calls received by the answering service have a length that falls between the lower limit and upper limit of the confidence interval you calculated in part (a.) above.

(8) False: [We are 95% confident that the average falls between these two limits - but this does not mean that 95% of the calls received fall between these limits.]

$\boxed{16}$ d.(4 points) The manager of the answering service collects the billing rate for *all* 472 clients during a month and finds the average bill to be $582.45 with a standard deviation of $231.59. If appropriate, calculate a 95% confidence interval for the average monthly bill for the answering service. If this is not appropriate, clearly state why not.

(12) Not appropriate: This is the entire population. (4)

4→ $\boxed{16}$ 5. (4 points) To estimate the percentage of people who approve of Presidents Bush's visit to Baghdad on Thanksgiving, a major newspaper plans randomly sampled 500 people from Salt Lake City and 500 people from Logan. All other things being equal:

(12) a. The accuracy in Logan will be about the same as the accuracy in Salt Lake City.
b. The accuracy in Logan will be quite a bit less than the accuracy in Salt Lake City
c. The accuracy in Logan will be quite a bit greater than the accuracy in Salt Lake City.
(Choose one options and explain briefly).

a): The accuracy depends on the size of the sample and not on the size of the population. (4)

15 → 60 6. Psychologists have long studied anxiety…you know, that feeling you get just before taking a statistics final. Their research has led them to distinguish between just-before-the-final anxiety from a more general state in which some people are just more anxious than others. The former is called *state* anxiety – produced by a particular environment condition such as a statistics test. The latter is called *trait* anxiety, a condition that generalizes across situations. The following data was collected from a random sample of college sophomores:

(X) - average State Anxiety: 7.3, SD = 2.37
(Y) - average Trait Anxiety: 6.6, SD = 2.07
r = 0.80

20 a. (5 points) Find the equation of the regression line for predicting Trait Anxiety from State Anxiety.

$$\text{slope} = r \cdot \frac{SD_Y}{SD_X} = 0.8 \cdot \frac{2.07}{2.37} = 0.699 \approx 0.7 \quad (8)$$

$$\text{intercept} = \text{avg}_Y - \text{slope} \cdot \text{avg}_X = 6.6 - 0.7 \cdot 7.3 = 6.6 - 5.11 \approx 1.5 \quad (8)$$

regression equation: $\boxed{y = 1.5 + 0.7 \cdot x}$ or $\boxed{\text{trait anx.} = 1.5 + 0.7 \cdot \text{state anx.}}$ (4)

16 b. (4 points) Predict the Trait Anxiety of a sophomore with a State Anxiety score of 9.4.

for state anx. = 9.4:

$$\text{trait anx.} = 1.5 + 0.7 \cdot 9.4 = 1.5 + 6.58 \approx 8.1 \quad (16)$$

12 c. (3 points) About how far off do you expect your prediction to be?

$$\text{r.m.s. error} = \sqrt{1-r^2} \cdot SD_Y = \sqrt{1-0.8^2} \cdot 2.07 = \sqrt{0.36} \cdot 2.07$$
$$= 0.6 \cdot 2.07 = 1.24 \quad (12)$$

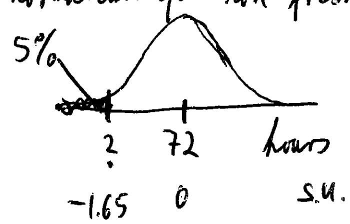
12 d. (3 points) Would you be surprised to learn that a sophomore, with a *state* anxiety score of 9.4, had a *trait* anxiety score of 4.6? Explain your reasoning, using the rms error score.

S.U.: $\frac{4.6 - 8.1}{1.24} = \frac{-3.5}{1.24} = -2.82$ (4) → we would be somewhat surprised: (4)
A trait anxiety score of 4.6 is about 2.8 r.m.s errors below the predicted trait anxiety score of 8.1. Only about 0.25% of all people with a state anxiety score of 9.4 will have such a low trait anxiety score (or lower). (4)

6 → 24 7. (6 points) A certain brand of wheat bread can be classified as "fresh" for 72 hours with a SD of 8 hours. (We will assume that "freshness" follows the normal curve). The bakery wants 95% of the bread to be "fresh" if it is on the display shelf. How many hours after baking should the bread be taken off of the shelf? (Hint: what *percentile* are we looking for?)

(4)
We are looking for the 5th percentile. (After 72 hours, 50% of the bread isn't fresh any more!)

normal curve for "non-fresh" bread



(8)
area between −1.65 and 1.65: 90.11% ≈ 90%

area below −1.65: 5% (4)

original units: $-1.65 \cdot 8 + 72 = 58.8$ hours (8)

10 → 40

8. The data shown in the table below were collected in a survey of the relationship between average number of hours spent studying per day and performance in advanced chemistry courses at a particular university. Determine whether there is a relationship between study time and course performance.

32 a. (State the null and alternative hypothesis, perform the appropriate statistical test, and clearly state your conclusions. (8 points)

−24 for incorrect test
−3 if null, alt swapped

Hours of Study per day	Performance Pass	Fail	Total
2 or less	60	32	92
3	65	13	78
4	25	5	30
Total	**150**	**50**	**200**

expected:

69	23
58.5	19.5
22.5	7.5

6×2 = 12

χ^2-test for independence:

1) null: study time and course performance are independent, (2)
i.e., boxes are identical (1)
alternative: study time and course performance are not (2)
independent (course performance depends on study time), i.e., at least one box is different (1)

2) expected: $\frac{150\cdot 92}{200} = 69$ etc. (see table "expected" above)

$$\chi^2 = \text{sum of } \frac{(\text{obs}-\text{exp})^2}{\text{exp}} = \frac{(60-69)^2}{69} + \frac{(32-23)^2}{23} + \frac{(65-58.5)^2}{58.5} + \frac{(13-19.5)^2}{19.5} + \frac{(25-22.5)^2}{22.5} + \frac{(5-7.5)^2}{7.5} = 8.7$$ (4)

$df = (3-1)\cdot(2-1) = 2$ (2)

3) $\chi^2 = 8.7$ between 5.99 and 9.21
5% 1%
→ P-value between 1% and 5% (4)

4) • reject the null (2)
• result is statistically significant
• course performance depends on study time (2)

8 b. (2 points) Would you classify your results as (a) non-significant, (b) statistically significant or (c) highly statistically significant? (circle your choice).

(8)

4 → 16

9. (4 points) In a certain county in Pennsylvania, it is known that exactly 75% of the adult residents have seen the movie "The Two Towers".

1. In a simple random sample of 1000 adult residents between 74% and 76% have seen the movie.
2. In a simple random sample of 100 adult residents between 74% and 76% have seen the movie.

Which of the following is more likely: (no explanation is necessary)

16 a. 1 is more likely than 2.
b. 2 is more likely than 1.
c. Both are equally likely.

a): The larger the sample size, the more likely it is that the sample % is very close to the population %, here 75%.

14 → $\boxed{56}$ 10. Suppose that we are drawing **four** times **without replacement** from a candy dish containing the following M & M candies: (each part worth 2 points)

10 brown
8 green
12 red
30 total

$\boxed{8}$ a. What is the probability that all four draws will be green M & M's?

1st green: $\frac{8}{30}$ (2) 2nd green, given 1st green: $\frac{7}{29}$ (1) 3rd green, given 1st & 2nd green: $\frac{6}{28}$ (1) 4th green, given 1st, 2nd & 3rd green: $\frac{5}{27}$ (1)

(3) for multiplication

Chance all 4 are green: $\frac{8}{30}\cdot\frac{7}{29}\cdot\frac{6}{28}\cdot\frac{5}{27} = 0.00255 \approx 0.26\%$

$\boxed{8}$ b. What is the probability that not all the candies you draw are green?

opposite of a): $100\% - 0.26\% = 99.74\%$ (8)

opposite rule

(8)

$\boxed{8}$ c. True or false (circle your choice) The events in part (b) above are independent.

[The color in the next draw depends on all previous draws.]

Now suppose that we draw once from the original candy dish described above (before we made any draws) and once from a candy dish containing the following:

6 green jelly beans
7 yellow jelly beans
7 pink jelly beans
20 total

$\boxed{8}$ d. What is the probability of getting a red M&M and a pink jelly bean?

red M&M: $\frac{12}{30}$ pink jelly bean: $\frac{7}{20}$

(2) (3) (3)

red M&M and pink jelly bean: $\frac{12}{30}\cdot\frac{7}{20} = \frac{84}{600} = 0.14 = 14\%$

multiplication rule

(8)

$\boxed{8}$ e. True or false (circle your choice) The events in part (d) above are independent.

[The chance of getting a particular jelly bean does not change whatever M&M we have drawn.]

$\boxed{8}$ f. Suppose we draw two cards from a well-shuffled standard deck of cards. What is the probability of getting the King of Hearts on the first draw or the King of Hearts on the second draw. (There are 52 cards in a standard deck – one King of Hearts).

These are mutually exclusive (see g) below), i.e. they cannot happen together.

KoH on first or KoH on second draw: $\frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26} = 0.038 = 3.8\%$

(2) (3) (3) addition rule

(8)

$\boxed{8}$ g. True or False (circle your choice) The events in part (f) above are mutually exclusive.

more detailed explanation:

[KoH on first, but not on second: $\frac{1}{52}\cdot\frac{51}{51} = \frac{1}{52}$

not KoH on first, but KoH on second: $\frac{51}{52}\cdot\frac{1}{51} = \frac{1}{52}$]

clearly mutually exclusive

KoH on first or second: $\frac{1}{52} + \frac{1}{52} = 3.8\%$