Question 1: Chances and Probabilities (30 Points)

I have a bag with 20 balls in it: 10 are red, 8 are blue, and 2 are green.

1. If I draw one ball at random from the bag, what is the chance that I get a red ball or a green ball? (10 Points)
   
   \[ \text{Chance of red ball: } \frac{10}{20} = \frac{1}{2} = 50\% \]
   \[ \text{Chance of green ball: } \frac{2}{20} = \frac{1}{10} = 10\% \]
   
   \[ \text{Total chance: } \frac{12}{20} = \frac{3}{5} = 60\% \]

2. If I draw two balls at random without replacement, what is the chance that I get a red ball, followed by a green ball? (10 Points)
   
   \[ \text{Total chance for first ball: } \frac{10}{20} = \frac{1}{2} = 50\% \]
   \[ \text{Total chance for second ball, given first is red: } \frac{9}{19} = 10.5\% \]
   \[ \text{Total chance for red and green: } \frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = 0.6526 = 63.7\% \]

3. If I draw three balls at random with replacement, what is the chance that I get at least one red ball? (10 Points)
   
   \[ \text{Total chance for ball: } \frac{10}{20} = \frac{1}{2} \]
   \[ \text{Total chance for no red balls: } 1 - \frac{1}{2} = \frac{1}{2} \]
   \[ \text{Total chance for at least one red ball: } 1 - \frac{1}{2} = \frac{1}{2} = 50\% \]

Question 2: Regression (50 Points)

A student wonders if people of similar heights tend to date each other. To find this out, she measured herself, her dormitory roommates, and all of her female classmates; then she measured the men each woman was currently dating. After making many measurements and analyzing the data, she found out that the women were on average 66 inches tall, with a standard deviation of 2.0 inches; their dates were on average 69 inches tall, with a standard deviation of 2.5 inches. The correlation coefficient between the women's and the men's heights was 0.57.

1. Find the regression equation for predicting the height of a woman’s date based on her own height. (20 Points)

   \[ r = \frac{0.57}{2} \approx 0.71 \]

2. Using your regression equation, predict the height of a date for a woman who is 67 inches tall. (10 Points)

3. Find the r.m.s. error for predicting the date's height from the woman's height. (10 Points)

4. Does the slope of the regression line (that you found in Part 1) say that men, if they date taller women, will become taller? Why or why not? Explain! (10 Points)
Question 3: Sampling (30 Points)

In Web polls, anyone who views a certain Web page is allowed to vote by clicking on their choice of button. In fact, there is nothing to stop someone from voting as many times as they want. The results of one such poll suggest that almost 90% of the US population wants to ban firearm sales. The poll has a very large sample size (over 1 million).

1. Web based polls such as this are notoriously susceptible to bias. Give three possible sources of bias for this poll. (21 Points)

\[1 \times (1) \quad 3 \times (3) \quad 90 \times (9) \] or \[1 \times (1) \quad 90 \times (9) \] etc.

number of dies: 500

2. How many people in our sample do you expect to have the blood type AB? (10 Points)

\[\text{Expected: } 500 \times \frac{1}{10} = 50 \]

3. What is the corresponding standard error? (10 Points)

\[\text{SE}_{\text{thr}} = \sqrt{500 \times 0.3} = 22.36 \approx 6.7 \]

4. What is the chance that fewer than 40 people in our sample have the blood type AB? (20 Points)

\[40 \text{ lower limit} - 15 \text{ upper limit} = 65 \text{ lower limit} - 15 = 50 \text{ upper limit} \]

\[z = \frac{40 - 50}{6.7} = -1.49 \]

\[\text{Correct z-score} = -1.49 \text{ (correct)} \]

\[\text{Correct s.e. calculation} = -1.49 \text{ (correct)} \]

\[\text{Correct table value} = -1.49 \text{ (correct)} \]

\[\text{Correct raw} = -1.49 \text{ (correct)} \]

i.e. chance that fewer than 40 people in sample have blood type AB is 6.68%
Question 5: Law of Averages (40 Points)

A die will be rolled some number of times. Which is better: 60 rolls or 600 rolls in the situations listed below? Circle the number and briefly explain your answer for each situation:

1. You win $1 if it shows □ more than 20% of the times. 60 / 600 (10 Points)
   - Correct number
   - Correct explanation
   - Incorrect number
   - Incorrect explanation

   Workbk answer:
   "To win, you need a large percentage error, and that is more likely in 60 rolls."

2. You win the dollar if the percentage of □'s is more than 15%. 60 / 600 (10 Points)
   - Workbk answer:
   "You want a small percentage error."

3. You win the dollar if the percentage of □'s is between 15% and 20%. 60 / 600 (10 Points)
   - Workbk answer:
   "You want a small percentage error."

4. You win the dollar if the percentage of □'s is exactly 16%. (60) / 600 (10 Points)
   - Workbk answer:
   "10 rolls because to get exactly the expected value means getting exactly 10 chance error, and that is more likely with more rolls."

Question 5:
Additional explanation:

Although only a verbal explanation was required (e.g., 96% is a better chance of success, hence more likely to occur), we can answer the question more accurately using a box model and working with EV of 0.5 and SE = 0.3:

For 60 draws:
- EV = 60 * 0.5 = 30
- SE = \sqrt{60} * 0.37 = 4.78
- EV% = 50% < the same > EV% = 50%
- SE% = \frac{4.78}{60} * 100% = 7.97%

For 600 draws:
- EV = 600 * 0.5 = 300
- SE = \sqrt{600} * 0.37 = 9.06
- EV% = 50% < the same > EV% = 50%
- SE% = \frac{9.06}{600} * 100% = 1.51%

Notice SE% is smaller for 600 draws than for 60 draws. Thus we will be closer to EV% = 50% after 600 draws than after 60 draws.

We also can calculate the exact chance (e.g., more than 2%) of "1" for more than 15% of "1"s) using the normal curve.