Math and Stat Colloquium

Tuesday, Nov. 27

3 p.m. MAIN 201

Refreshments will be served in the Animal Science Lounge at 2:30 p.m.

Speaker:

Alexander Guterman

Moscow State University

"Polya permanent problem: 99 years after"

This is a joint work with Mikhail Budrevich. Gregor Dolinar, Bojan Kuzma and Marko Orel.

Two important functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad \text{and} \quad \mathrm{per\,A} = \sum_{\sigma \in \mathrm{S_n}} \mathrm{a_{1\sigma(1)} \cdots a_{n\sigma(n)}}$$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix and S_n denotes the set of all permutations of the set $\{1, \ldots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent. Due to this reason, starting from the work by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation.

A transformation T on a certain matrix set S is called a *converter on* S if $\mathrm{per\,A} = \det \mathrm{T(A)}$ for all $A \in S$. A single matrix A is called *sign-convertible* if there exists a $(+1, -1)$ matrix X such that $\mathrm{per\,A} = \det(\mathrm{X} \circ \mathrm{A})$, where $X \circ A$ is the entrywise product of matrices.

Among our results we prove the following theorem:

Theorem 1. *Suppose* $n \geq 3$, *and let* $\mathbb{F}$ *be a finite field with* $\mathrm{char}\,\mathbb{F} \neq 2$. *Then, no bijective map* $T : M_n(\mathbb{F}) \to M_n(\mathbb{F})$ *satisfies*

$$\mathrm{per\,A} = \det \mathrm{T(A)}.$$

Also we investigate Gibson barriers (the maximal and minimal numbers of non-zero elements) for convertible $(0, 1)$-matrices and solve several related problems on different matrix subspaces.

Our results are illustrated by the number of examples.