Math and Stat Colloquium

Thursday, Dec. 2

3:30pm BUS 116

Refreshments will be served in the Lund Hall Foyer at 3 pm

Speaker:

David E. Brown

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"Riemann's Zeta At Even Positive Integers"

ABSTRACT. *The Riemann Hypothesis* (RH) is the oldest of the seven Millennium Problems and, being a member of this group of problems, it enjoys a \$1,000,000 bounty. RH is a conjecture about the zeros of the analytic continuation of the function $\zeta(s) = \sum_{n\geq 1} \frac{1}{n^s}$, called the *Riemann Zeta* function. In this talk, I intend to give a description of RH with the following promises: (1) It will illustrate the importance of RH with respect to Mathematics and the world writ large; (2) It will make clear the connections between the distribution of primes and the solution of RH; (3) It will be understandable to a high school student.

I will also get my hands dirty with the Riemann Zeta function and calculate $\zeta(s)$ for s an even positive integer. Exact calculation of $\zeta(s)$, where s is an odd positive integer, is an open problem. I will present one of Euler's non-rigorous but ingenious (of course) solutions to the *Basel Problem* (equivalently: evaluate $\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$) which challenged many Mathematicians from 1644 to 1735. Historians believe that Johann Bernoulli shared the Basel Problem with Euler. Johann Bernoulli was Euler's teacher (by today's standards he would have been Euler's PhD advisor) and brother of Jakob Bernoulli who studied sums of the form $S_m(n) = 0^m + 1^m + 2^m + 3^m + \cdots + (n-1)^m$. From this study he developed the *Bernoulli Numbers*. If B_n denotes the nth Bernoulli number, I will show that $\zeta(2k) = \frac{(-1)^k \pi^{2k} 2^{2k-1} B_{2k}}{(2k)!}$, where k is a positive integer.