Activity Plan: Confidence Intervals

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Topic: Confidence Intervals

Overview: Students simulate drawing colored balls from a jar to investigate the interpretation of confidence intervals and the effect on the intervals of changing the confidence level or sample size.

Objectives:
1. Students will understand that a 95% (90%, 99%) CI means that approximately 95 (90, 99) out of 100 CI's at that confidence level will cover the population proportion.
2. Students will understand that increasing sample size (all else being equal) results in narrower confidence intervals and will offer an explanation for this result.
3. Students will explain that increasing confidence level (all else being equal) results in wider confidence intervals and will offer an explanation for this result.

Technology: Confidence Intervals Applet: https://math.usu.edu/~schneit/Statlets/CI/index.html

The Role of Technology: The applet facilitates investigation of effect of sample size and confidence level on the width of a confidence interval and motivates a frequentist interpretation.

Applet Overview: The large 'jar' contains 10,000 blue and orange balls. What proportion of the balls are blue?

To know the true proportion of blue balls, it would be necessary to count all the blue balls in the large jar. A large population like this one is often inaccessible due to constraints of time and other resources. Instead, we choose a (much smaller) sample from the jar, compute the proportion of blue balls in the sample, and use that to estimate the proportion of blue balls in the whole population.

Buttons:

- Sample – Use this button to choose a single sample from the large jar. The sample will appear in the small jar and a confidence interval for the proportion of blue balls in the population, based on the sample, is drawn on the plot below. Repeat this process to draw more samples.
- Sample 100 – This button will cause the applet to choose 100 samples from the large jar in rapid succession. Only the last of the 100 will be shown in the sample jar. Confidence intervals for the proportion of blue balls in the population are plotted on the graph below.
- Clear plot – This button erases the graph and empties the sample jar but does not change the population. Use this button when you want to look at confidence intervals based on a new sample size.
- New Population – This button clears the plot, empties the sample jar, and fills the large jar with a new population of balls.

Other components:

- Show p check box – Use this box to show the value of the parameter, that is, the true proportion of blue balls in the population (large jar). The proportion is displayed beside the check box and is indicated in the plot.
- Sample size radio buttons – Use the radio buttons to set the size of the sample: 50, 100 (default), or 250 balls.
- Confidence radio buttons – The radio buttons can be used to set the confidence level to 90, 95 (default), or 99.

Confidence Intervals Task Sheet

Applet: https://math.usu.edu/~schneit/Statlets/CI/index.html

1. Click 'Sample' to choose one sample from the population. The sample is shown in the small jar and a confidence interval based on that sample is plotted in the graph. Where is the middle of the interval? About how wide is the interval?

2. Use the 'Sample 100' button to general 100 samples and plot a CI based on each. Why are the intervals not all the same?

3. Are there any intervals that don't overlap? What does this mean? How is this possible?

4. Click the 'Show p' box to see the true proportion of blue balls in the population. This proportion will be drawn in the graph as a blue horizontal line. Some of the confidence intervals are now drawn in red.

- What does red indicate about an interval?
- About how many of the 100 95% confidence intervals are red?

5. Clear the plot, then generate 100 more intervals. Show the value of the parameter. How many of the intervals are red? Do this a couple more times, generating 100 intervals and counting the number of reds.

6. Change the Confidence level to 90, now how many out of the 100 are red?

7. How many red intervals would you expect to see if you change the confidence level to 99? Explain.

8. For 100 95% (or 90% or 99%) confidence intervals, will the number of red intervals always be the same? Explain.

9. Can you construct a 100% confidence interval for the proportion of blue balls in the population? Explain.

10. How does increasing the confidence level affect the width of the confidence interval?

11. The applet will also allow you to change the sample size used in constructing a CI.
Conjecture: How will the CI change if the sample size is increased? Explain.

12. Increase the sample size to 250 and use the ‘Sample 100’ button to generate 100 new CI’s.