Chapter 9.3 Homework Page

Exercises 1-4 Solve, Draw Graphs *Find all pairs of real numbers x, y that satisfy the system of equations. Draw graphs and show points of intersection (if any).*

1. $2x - y = -2$
$xy = 4$

Solution:

From E_1, $y = 2x + 2$; substitute into E_2. $x(2x + 2) = 4$, $x^2 + x - 2 = 0$, $(x + 2)(x - 1) = 0$, $x = -2$ or $x = 1$. When $x = -2$, $y = 2(-2) + 2 = -2$. When $x = 1$, $y = 2 \cdot 1 + 2 = 4$. From graphs we see that the line and the graph of $xy = 4$ intersect at points $(-2, -2)$ and $(1, 4)$.

2. $x + y = 2$
$x^2 + y^2 = 2$

Solution:

From the first equation, $x = 2 - y$; substitute into the second equation. $(2 - y)^2 + y^2 = 2$, $4 - 4y + y^2 + y^2 = 2$, $y^2 - 2y + 1 = 0$, $(y - 1)^2 = 0$, $y = 1$. When $y = 1$, $x = 2 - 1 = 1$. Draw graphs and see that the line and circle just touch (are tangent) at the point $(1, 1)$.

3. $y = x^2 - 4x + 4$
$y = -2x^2 + x + 16$

Solution:

Eliminating y, we get $x^2 - 4x + 4 = -2x^2 + x + 16$, or $3x^2 - 5x - 12 = 0$, $(3x + 4)(x - 3) = 0$, $x = -4/3$ or $x = 3$. When $x = -4/3$, $y = 100/9$. When $x = 3$, $y = 1$.
The graphs are two parabolas that intersect at $(-4/3, 100/9)$ and $(3, 1)$.

4. $2x - y = 0$
$xy - y = 2$

Solution:

From E_1, $y = 2x$; substitute into E_2. $x(2x) - 2x = 2$, $x^2 - x - 1 = 0$, $x = \left(1 \pm \sqrt{5}\right)/2$. When $x_1 = \left(1 + \sqrt{5}\right)/2 \approx 1.62$, $y = 2x_1 = 1 + \sqrt{5} \approx 3.24$. When $x_2 = \left(1 - \sqrt{5}\right)/2 \approx -0.62$, $y = 2x_2 = 1 - \sqrt{5} \approx -1.24$. Draw graphs and see that the line and the rational function, $y = 2/(x - 1)$, intersect at approximately $(1.62, 3.24)$ and $(-0.62, -1.24)$.

Exercises 5-6 Nonlinear Systems *Solve the system of equations. If results involve irrational numbers, give approximations rounded off to two decimal places.*

5. $y = e^x$
$x + \ln y = 0$

Solution:

From E_2, $x = -\ln y$; substitute into E_1. $y = e^{-\ln y}$, $y = e^{\ln y^{-1}}$, $y = y^{-1}$, $y = 1/y$, $y^2 = 1$. Therefore, $y = 1$ or $y = -1$. When $y = 1$, $x = -\ln 1 = 0$. When $y = -1$, $x = -\ln(-1)$, undefined. Therefore, there is only one solution, $(0, 1)$.

6. $3^x + 3y = 10$
$3^{x-1} - y = 8$

Solution:

From E_2, $y = 3^{x-1} - 8$; substitute into E_1. $3^x + 3(3^{x-1} - 8) = 10$, $3^x + 3^x - 24 = 10$, $2 \cdot 3^x = 34$, $3^x = 17$, $\ln 3^x = \ln 17$, $x \ln 3 = \ln 17$, $x = \ln 17/\ln 3 \approx 2.58$. When $x \approx 2.58$, $y = 3^{x-1} - 8 \approx 3^{1.58} - 8 \approx -2.33$. Therefore, there is only one solution, $(2.58, -2.33)$.

Exercises 7 Rectangles

7. Find the dimensions of a rectangle that has a diagonal of length 13 cm and a perimeter of 34 cm.

Solution:

Let x, y be the dimensions of the rectangle with diagonal 13 cm: $\sqrt{x^2 + y^2} = 13$. Perimeter is 34 cm: $2x + 2y = 34$. Therefore, we want solutions to the system of equations $x^2 + y^2 = 169$, $x + y = 17$. Solving we get $x = 5$, $y = 12$ or $x = 12$, $y = 5$. Therefore, the rectangle is 5 cm $\times$ 12 cm.