Math 5710
Continuous Random Variables

1. Let \[ f(x) = cx^2 \] for \( 0 < x < 2 \).
   a) Find the value of \( c \) that makes \( f(x) \) a density function for a random variable \( X \).
   b) Find the cumulative distribution function for \( X \).
   c) Find \( P(X^2 < 2) \)

2. When a certain component of a manufacturing process breaks down, the time that it takes to fix it (in hours) is a random variable with the density function

\[
  f(x) = \begin{cases} 
  ce^{-3x} & \text{if } 0 \leq x < \infty \\
  0 & \text{otherwise.}
  \end{cases}
\]

   a) Calculate the value of \( c \).
   b) Find the probability that, when this component breaks down, it takes at most 2 hours to fix it.