

**Set Theory Axioms:**      *Naive Set Theory by Paul R. Halmos*

- 1. Axiom of extension.** Two sets are equal if and only if they have the same elements.
  
- 2. Axiom of unions.** For every collection of sets there exists a set that contains all the elements that belong to at least one set of the given collection.
  
- 3. Axiom of specification.** To every set  $A$  and to every condition  $S(x)$  there corresponds a set  $B$  whose elements are exactly those elements  $x$  of  $A$  for which  $S(x)$  holds.
  
- 4. Axiom of pairing.** For any two sets there exists a set that they both belong to.
  
- 5. Axiom of powers.** For each set there exists a collection of sets that contains among its elements all the subsets of the given set.
  
- 6. Axiom of infinity.** There exists a set containing  $0$  and the successor of each of its elements.
  
- 7. Axiom of substitution.** If  $S(a,b)$  is a sentence such that for each  $a$  in set  $A$  the set  $\{b: S(a,b)\}$  can be formed, then there exists a function  $F$  with domain  $A$  such that  $F(a) = \{b: S(a,b)\}$  for each  $a$  in  $A$ . (Anything intelligent that one can do to the elements of a set yields a set.)
  
- 8. Axiom of choice.** The Cartesian product of a non-empty family of non-empty sets is non-empty.