1. For each set below, determine whether or not it is bounded above or bounded below or neither. Where appropriate, determine the supremum, the infimum, the maximum, and the minimum of the set.

   a) \( \{ \frac{1}{n} : n \in J \} \)  
   b) \( \{ r \in Q : r^2 < 4 \} \)

   c) \( \{ r \in Q : r^2 < 2 \} \)  
   d) \( \bigcap_{n=1}^{\infty} (1 - \frac{1}{n}, 1 + \frac{1}{n}) \)

   e) \( \bigcap_{n=1}^{\infty} [ -\frac{1}{n}, 1 + \frac{1}{n} ] \)  
   f) \( \{ 3r : r \text{ is irrational} \} \)

2. Restate the completeness axiom in terms of greatest lower bounds, and prove the equivalence of the two forms.

3. Suppose \( S \) is a non empty set which is bounded below. Show that for each \( \epsilon > 0 \), there exists \( x \) in \( S \) such that \( \inf S \leq x < \inf S + \epsilon \). (This is the back away principle for infimums.)

4. If \( A \) and \( B \) are bounded sets and \( A \subseteq B \), show that \( \sup A \leq \sup B \) and \( \inf A \geq \inf B \). If \( \sup A = \sup B \) and \( \inf A = \inf B \), must \( A = B \)?

5. Prove that for every pair of positive real numbers, \( x \) and \( y \), there is an \( n \) in \( J \) such that \( y < n \cdot x \).

6. Show that for every positive real number \( x \), there is an \( n \) in \( J \) such that \( 0 < \frac{1}{n} < x \).

7. Show that for every pair of positive real numbers \( x \) and \( y \), there exists an \( n \) in \( J \) such that \( 0 < \frac{2}{n} < x \).

8. Prove that between any two distinct real numbers lies a rational number.

9. Prove that between any two distinct real numbers lies an irrational number.