Math 4200

Definitions.

Let $S$ be a nonempty subset of the real line $R$.

1. If $S$ contains a largest element $w$ (i.e., $w$ is an element of $S$ and $s \leq w$ for all $s \in S$), then $w$ is called the maximum of $S$, $w = \max S$.

2. If $S$ contains a smallest element $v$ (i.e., $v$ is an element of $S$ and $s \geq v$ for all $s \in S$), then $v$ is called the minimum of $S$, $v = \min S$.

3. If $M$ is an element of $R$ and $s \leq M$ for all $s \in S$, then $M$ is called an upper bound of $S$ and the set $S$ is said to be bounded above.

4. If $m$ is an element of $R$ and $s \geq m$ for all $s \in S$, then $m$ is called a lower bound of $S$ and the set $S$ is said to be bounded below.

5. The set $S$ is said to be bounded if it is bounded above and bounded below.

6. If $S$ is bounded above and $S$ has a least upper bound, then it is called the supremum of $S$ and denoted by $\sup S$.

7. If $S$ is bounded below and $S$ has a greatest lower bound, then it is called the infimum of $S$ and denoted by $\inf S$.

Completeness Axiom.

Every nonempty subset $S$ of $R$ that is bounded above has a least upper bound.