**Definition.** A sequence \( \{a_n\} \) is said to be a Cauchy sequence if for each \( \epsilon > 0 \), there exists \( N > 0 \) such that \( |a_n - a_m| < \epsilon \) whenever \( n, m > N \).

**Theorem.** Every Cauchy sequence of real numbers is bounded.

**Proof.**

Let \( \{a_n\} \) be a Cauchy sequence. Let \( \epsilon = 1776 \). There exists \( N > 0 \) such that \( |a_n - a_m| < \epsilon \) whenever \( n, m > N \). Let \( k > N \). If \( n > k \), then \( |a_n - a_k| < 1776 \). That is, if \( n > k \), \( a - 1776 < a_n < a_k + 1776 \). It also follows that \( |a_n| < \max\{|a_k + 1776|, |a_k - 1776|\} \) for \( n > k \).

Let \( B = \max\{|a_1|, |a_2|, ..., |a_k|, |a_k + 1776|, |a_k - 1776|\} \). Since \( |a_n| \leq B \) for all \( n \), \( B \) is a bound for the sequence \( \{a_n\} \).

**Theorem.** A sequence \( \{a_n\} \) is a convergent sequence if and only if it is a Cauchy sequence.