Riemann Integral:

Let \( f \) be a bounded function on \([a,b]\).

Let \( P: a = x_0 < x_1 < \ldots < x_n = b \) be a partition of \([a,b]\). The norm of the partition, denoted \(|P|\), is equal to \( \max \{x_i - x_{i-1} : i = 1, 2, \ldots, n\} \).

For \( i = 1, 2, \ldots, n \) let \( c_i \in [x_{i-1}, x_i] \).

\[
\sum_{i=1}^{n} f(c_i) (x_i - x_{i-1})
\]
is called a Riemann Sum for \( f \) over \([a, b]\).

The function \( f \) is said to be Riemann integrable on the interval \([a, b]\) provided

\[
\lim_{|P| \to 0} \sum_{i=1}^{n} f(c_i) (x_i - x_{i-1}) = L.
\]

That is, for each \( \varepsilon > 0 \) there exists \( \delta > 0 \) such that given any partition \( P: a = x_0 < x_1 < \ldots < x_n = b \) with \(|P| < \delta\), and given any choice of \( c_i \in [x_{i-1}, x_i], i = 1, 2, \ldots, n \), it follows that

\[
\left| \sum_{i=1}^{n} f(c_i) (x_i - x_{i-1}) - L \right| < \varepsilon.
\]

Notation: The limit is usually denoted by \( \int_{a}^{b} f(x)dx \).