Solving Applied Problems

1. Understand.
2. Choose variable (variables).
3. Obtain a mathematical representation of the problem.
4. Do the math.
5. Interpret your results.

A small cruise ship that can hold up to 60 people provides three-day excursions to groups of 40 or more. If the group contains 40 people, each person pays $70. The cost per person for all members of the party is reduced by $1 for each person in excess of 40. Find the size of the group that maximizes income.

Let $x =$ number of additonal people, $1 \le x \le 20$. Now $40 + x =$ total number of passengers.

Then $70 - x =$ ticket price per passenger.

The income or total revenue is $R(x) = (40 + x)(70 - x)$.

Find the value of x for which the function $R(x)$ is a maximum.

The function $R(x) = (40 + x)(70 - x) = -x^2 + 30x + 2800$ is a quadratic function.

Recall that a quadratic function $f(x) = ax^2 + bx + c$ has a maximum value when $a < 0$ and $x = \dfrac{-b}{2a}$.

The function $R(x) = (40 + x)(70 - x) = -x^2 + 30x + 2800$ is a maximum when $x = 15$.

A group size of 55 people will maximize income.