

# Properties of Logarithms

## Properties of Logarithms

If  $b$  is a positive real number then the following relationships exist:

1.  $\log_b 1 = 0$
2.  $\log_b b = 1$
3.  $\log_b b^x = x$
4.  $b^{\log_b x} = x$  for  $x > 0$  (Why do we have the limitation  $x > 0$ ?)
5.  $\log_b(MN) = \log_b M + \log_b N$  for  $M > 0$  and  $N > 0$
6.  $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$  for  $M > 0$  and  $N > 0$
7.  $\log_b(M^p) = p \log_b M$  for  $M > 0$

Simplify the following logarithmic expressions without using a calculator:

$$\log_2 8\sqrt{2}$$

$$\log_5 \frac{\sqrt[3]{5}}{25}$$

$$\log_9 3 + \log_9 27$$

$$e^{-4 \ln 2 + \ln 3}$$

Use the properties of logarithms to write the following as a logarithm of a single quantity:

$$4 \log_2 (xy^2) + 2 \log_2 \left(\frac{1}{x}\right) - 4 \log_2 y$$

$$\ln 7 + 6 \ln x - 6 \ln y$$

Expand the following logarithms:

$$\log_5(50x + 25y)$$

$$\log_2(8x^2 + 80x + 200)$$

$$\log(\log(10000^{250x}))$$

## The Change of Base Formula:

If  $b$ ,  $c$ , and  $x$  are positive real numbers then:

$$\log_b x = \frac{\log_c x}{\log_c b}$$

Give a 4 decimal approximation for  $\log_3 20$ .

Give a four decimal approximate for the number  $\log_{\frac{1}{7}}(0.719)$

On your calculator, graph the function  $f(x) = 2 - \log_3(x + 1)$

The **pH** of a solution is defined to be  $-\log([H_3O^+])$ , where  $[H_3O^+]$  is the concentration of hydronium ions in moles/liter. Solutions with a pH less than 7 are said to be *acidic*, while those with a pH greater than 7 are *basic*.

Example:

If a sample of a solution has a  $[H_3O^+]$  concentration of  $7.49 \times 10^{-6}$  moles/liter. What is the pH?

If  $I_0$  is the minimum discernable intensity of an earthquake, then an earthquake with an intensity of  $I$  has a **Richter Scale** ranking of:

$$R = \log\left(\frac{I}{I_0}\right)$$

If  $I_0$  is the minimum discernable intensity of a sound, then a sound with an intensity of  $I$  has a **decibel level** of:

$$D = 10 \log\left(\frac{I}{I_0}\right)$$

Example:

Given that  $I_0 = 10^{-12}$  watts/meter<sup>2</sup>. What is the intensity of a sound for which the decibel level is 102?