Locating Real Zeros of a Polynomials

The Factor Theorem:
If c is a zero of the polynomial $p(x)$ then $(x - c)$ is a factor of $p(x)$.

The Rational Zeros Theorem:
If the rational number $\frac{p}{q}$ is a zero of a polynomial, then p is a factor of the constant term and q is a factor of the leading coefficient.

Example:
Write out a list of the possible rational zeros for the polynomials given below:

$f(x) = x^3 - 7x^2 - 4x + 28$

$g(x) = 2x^4 - 3x^3 - 6x^2 + 6x + 4$

Descartes Rule of Signs:
If the polynomial function $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ has real coefficients and $a_0 \neq 0$ then:

-The number of positive real zeros is less than (by an even integer) or equal to the number of sign variations of $P(x)$.

-The number of negative real zeros is less than (by an even integer) or equal to the number of sign variations of $P(-x)$.

Use Descartes Rule of Signs to determine the possible number of positive real zeros and negative real zeros for the following polynomial functions.

$f(x) = 7x^3 - 2x^2 + 2x + 3$

$g(x) = \frac{1}{2}x^6 - \frac{1}{10}x^4 - \frac{1}{5}x^2 - \frac{1}{3}$

Upper and Lower Bounds of Real Zeros:

If $f(x)$ is a polynomial of degree $n \geq 1$ with real coefficients and a positive leading coefficient.

1) No real zero of f is larger than b if the last row in the synthetic division of $f(x)$ by $x - b$ contains no negative numbers. If all of the real zeros of f are less than b then b is said to be an **upper bound** of the zeros of f.

2) No real zero of f is smaller than a if the last row in the synthetic division of $f(x)$ by $x - a$ has entries that alternate in sign (0 can be counted as a either positive or negative). If all of the real zeros of f are greater than a, then a is said to be a **lower bound** of the zeros of f.

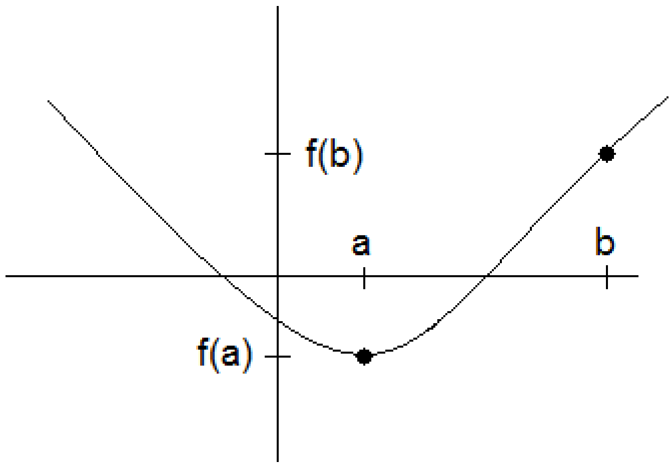
Determine the smallest integer that is an upper bound of the zeros of f and determine the largest integer that is a lower bound of the zeros of f.

$f(x) = 7x^3 - 2x^2 + 2x + 3$

Use Descartes' rule of signs, rational zeros test, upper and lower bounds test, and polynomial division to determine all of the zeros of the polynomial P.

$P(x) = x^3 - 3x^2 + 4x - 12$

The Intermediate Value Theorem:
Assume that $f(x)$ is a polynomial with real coefficients, and that a and b are real numbers with $a < b$. If $f(a)$ and $f(b)$ have different signs then there is at least one real zero c of f such that $a < c < b$.
See graph below:



Does the Intermediate Value Theorem guarantee that the polynomial function $f(x) = 2x^3 - x^2 - 3x$ has a zero in the interval $[1, 2]$?

Does the Intermediate Value Theorem guarantee that the polynomial function $g(x) = 4x^3 + 16x^2 + 5x - 25$ has a zero in the interval $[-3, -2]$?

If $-5/2$ is a zero of g, does that contradict the Intermediate Value Theorem?