Polynomial Division

Use the example below and review the algorithm for long division:

$$\left(2x^4 - x^3 + 7x^2 + 5\right) \div \left(2x^2 + 1\right)$$

Division of Polynomials:

Let $p(x)$ and $d(x)$ be polynomials such that $d(x) \neq 0$. Then there are unique polynomials $q(x)$ and $r(x)$, called the quotient and the remainder, respectively, such that

$$\frac{p(x)}{d(x)} \;=\; q(x) \;+\; \frac{r(x)}{d(x)}$$

or

$$p(x) \;=\; q(x)\, d(x) \;+\; r(x)$$

If $r(x) = 0$ then q and p are factors of the polynomial p

Use the rational number $\frac{212}{3}$ and division to understand the concepts of dividend, divisor, quotient, and remainder.

Use the rational expression $\frac{2x^4 \; - x^3 + 7x^2 + 5}{2x^2 + 1}$ and division to understand the concepts of dividend, divisor, quotient, and remainder.

Linear Factor Theorem:
The number k is a factor of a polynomial $p(x)$ if and only if the linear polynomial $x - k$ is a factor of p. That means, if k is a zero of $p(x)$ then:

1) $p(x) = (x - k)\,q(x)$ for some polynomial q.
2) $p(k) = 0$
3) k is an x-intercept of the graph of p.
4) If $p(x)$ is divided by $(x - k)$ then the remainder is $p(k)$.

Example: Show that 2 is a zero of the function $f(x) = x^3 - 2x - 4$
What other things can be determined about the function f based on this information?

Example: Show that $2 + \sqrt{3}$ is a zero of the function
$f(x) = x^2 - 4x + 1$
What other things can be determined about the function f based on this information?

Review synthetic division by dividing the following:

$$\frac{3x^3 - 2x^2 + 2}{x + 2}$$

Divide: $(2x^5 + x^4 - 18x^3 - 9x^2 + 16x + 8) \div (x + \frac{1}{2})$