Matrices and Gaussian Elimination

Definition:
A **matrix** of order $\boldsymbol{m \times n}$ is a rectangular array of m rows of real numbers and n columns of real numbers. A matrix is often labeled with a capital letter. The elements of a matrix are usually labeled with the same lower case letter used to identify the matrix with the pair of subscripts which refer to the row and column the element is found in. For example, the notation a_{ij} refers to the element of matrix A that is found in the ith row and jth column.

Example:

$$K = \begin{matrix} 2 & \frac{1}{2} & -1 \\ 1 & -1 & 0 \\ \frac{2}{3} & -5 & 12 \\ 0 & -2 & -10 \\ 9 & 4 & 0 \end{matrix}$$

What is the order of K?

What is $k_{3,1}$

What is $k_{5,2}$

What is $k_{2,5}$

We can use a matrix to streamline the process of solving a system of linear equations.

A matrix that is in **row-echelon** form has the following properties:

1) Any row consisting entirely of zeros occur at the bottom of the matrix.

2) For each row that does not consist entirely of zeros, the first nonzero entry is a 1 (called a leading 1).

3) The leading 1 in each row must lie farther to the right than the leading one in the row above it.

A matrix that is in **reduced row-echelon form** has the following properties:

1) Any row consisting entirely of zeros occur at the bottom of the matrix.

2) For each row that does not consist entirely of zeros, the first nonzero entry is a 1 (called a leading 1).

3) The leading 1 in each row must lie farther to the right than the leading one in the row above it.

4) Any <u>column</u> that contains a leading 1 has zeros every where else.

Determine which of the matrices below are in row-echelon form or Reduced Row-Echelon form.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Elementary Row Operations:
1) Interchange any two rows.
2) Multiply a row by a real number.
3) Replace a row with a sum of two rows.

Definition:
If a given matrix can be obtained from another matrix by performing elementary row operations then those matrices are said to be **row-equivalent.**

Theorem:
If a system of equations has an augmented matrix that is row equivalent to the augmented matrix from another system of equations then those systems have the same solution set.

Given the system of equations below, write the augmented matrix for the system and then use elementary row operations to convert the augmented matrix to row-echelon form in order to solve the system of equations. (Gaussian Elimination)

$$-3x+2y=4$$
$$-x-3y=-1$$

Given the system of equations below, write the augmented matrix for the system and then use elementary row operations to convert the augmented matrix to row-echelon form in order to solve the system of equations. (Gaussian Elimination)

$$\begin{aligned} 2x + 3y &= -4 \\ 3x - 5y &= 10 \end{aligned}$$

Solve the system:

$$\begin{aligned} x + 2y - z &= -5 \\ 3x + y + 2z &= 5 \\ -2x - 3y - 2z &= 3 \end{aligned}$$

Solve the system:

$$\begin{aligned} 2x - 3y + 2z &= -9 \\ -3x + 6y - 4z &= 19 \\ -x + y + 6z &= -4 \end{aligned}$$

Solve the system:

$$\begin{aligned} x + 2y &= -12 \\ x + 3z &= 8 \\ y + 5z &= 3 \end{aligned}$$

Solve the system:

$$\begin{aligned} 2x + y - 2z &= 6 \\ -4x - 3y + z &= -7 \\ 2x + 2y + z &= 1 \end{aligned}$$

Solve the system:

$$6x - 22y - 4z = 17$$
$$4y - z = -2$$
$$2x - 2y = 3$$

Solve the system:

$$\begin{aligned} 2x + z &= 3 \\ 2x - y + z &= -2 \\ 2x - 3y + z &= 10 \end{aligned}$$