

Reduce the following system to echelon form and find the solution set.

$$x - 2y + z = -3$$

$$x - y + 5z = -1$$

$$x - 3y - 3z = -5$$

- The matrix corresponding to this system of equations is $\begin{bmatrix} 1 & -2 & 1 & -3 \\ 1 & -1 & 5 & -1 \\ 1 & -3 & -3 & -5 \end{bmatrix}$

- $-R_1 + R_2 \rightarrow R_2$ $\begin{bmatrix} 1 & -2 & 1 & -3 \\ 0 & 1 & 4 & 2 \\ 1 & -3 & -3 & -5 \end{bmatrix}$ $-R_1 + R_3 \rightarrow R_3$ $\begin{bmatrix} 1 & -2 & 1 & -3 \\ 0 & 1 & 4 & 2 \\ 0 & -1 & -4 & -2 \end{bmatrix}$

- $R_2 + R_3 \rightarrow R_3$ $\begin{bmatrix} 1 & -2 & 1 & -3 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Now let $z = k$

- Using backward substitution $z = k$, $y + 4z = 2$, $y = 2 - 4k$, $x - 2y + z = -3$,
 $x = -3 + 2y - z$, $x = -3 + 2(2 - 4k) - k$,
 $x = 1 - 9k$

- The solution set is $\begin{matrix} x = 1 - 9k \\ y = 2 - 4k \\ z = k \end{matrix}$ where k is any real number.