

Reduce the following system to echelon form and find the solution set.

$$x+3y+z=12$$

$$-x+4y-z=9$$

$$2x+3y+z=10$$

- The matrix corresponding to this system of equations is $\begin{bmatrix} 1 & 3 & 1 & 12 \\ -1 & 4 & -1 & 9 \\ 2 & 3 & 1 & 10 \end{bmatrix}$.

- $R_1 + R_2 \rightarrow R_2$ $\begin{bmatrix} 1 & 3 & 1 & 12 \\ 0 & 7 & 0 & 21 \\ 2 & 3 & 1 & 10 \end{bmatrix}$ $-2R_1 + R_3 \rightarrow R_3$: $\begin{bmatrix} 1 & 3 & 1 & 12 \\ 0 & 7 & 0 & 21 \\ 0 & -3 & -1 & -14 \end{bmatrix}$

- $\frac{1}{7}R_2 \rightarrow R_2$: $\begin{bmatrix} 1 & 3 & 1 & 12 \\ 0 & 1 & 0 & 3 \\ 0 & -3 & -1 & -14 \end{bmatrix}$ $3R_2 + R_3 \rightarrow R_3$: $\begin{bmatrix} 1 & 3 & 1 & 12 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & -5 \end{bmatrix}$

- $-R_3 \rightarrow R_3$: $\begin{bmatrix} 1 & 3 & 1 & 12 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

- Using backward substitution $z=5$, $y=3$, $x+3y+z=12$,
 $x+3(3)+5=12$, and $x=-2$.