Find all of the zeros, real and imaginary, of the function $p(x) = x^3 - 5x - 2$.

Find all possible rational zeros: $\frac{\text{factors of } -2}{\text{factors of } 1}$

Possible rational zeros: $\pm 2, \pm 1$

$$p(1) = -6 \qquad p(-1) = 2$$

$$p(2) = -4 \qquad p(-2) = 0$$

If -2 is a zero of $x^3 - 5x - 2$ then $x + 2$ is a factor of $x^3 - 5x - 2$.

$$x^3 - 5x - 2 = (x + 2)(x - c_2)(x - c_3)$$

$$x^3 - 5x - 2 = (x+2)(x - c_2)(x - c_3)$$

Divide $x^3 - 5x - 2$ by $x + 2$

$$\begin{array}{r|cccc} -2 & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & \mathbf{0} \end{array}$$

$$x^3 - 5x - 2 = (x+2)(x^2 - 2x - 1)$$

Find the zeros of the quadratic factor

$$x^2 - 2x - 1 = 0$$

Find the zeros of the quadratic factor

$$x^2 - 2x - 1 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$1 - \sqrt{2} \text{ or } 1 + \sqrt{2}$$

$$\text{zeros of } x^3 - 5x - 2 : \{ -2, 1 - \sqrt{2}, 1 + \sqrt{2} \}$$

$$x^3 - 5x - 2 = (x + 2)\left(x - (1 - \sqrt{2})\right)\left(x - (1 + \sqrt{2})\right)$$