Utah State University
Department of Mathematics and Statistics

Math 5210-Math 5220
(Introduction to Analysis and Advanced Calculus)
Fall and Spring Semesters, 2004-2005

Textbook: The Elements of Real Analysis (2nd edition)
by Robert G. Bartle
John Wiley & Sons, publishers

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Office Hours: Tuesday: 1:30 pm - 2:30 pm
Wednesday: 10:00 am - 11:00 am
Thursday: 10:00 am -11:00 am
plus by appointment only; please make an appointment with me at least one day in advance if you cannot make a scheduled office hour.

Grading Policy: There will be two midterm examinations, a final **comprehensive** examination, and several written assignments to be turned in. Each midterm examination will be worth 20% of your final grade; the final examination is worth 25% of your final grade, and the written assignments will amount to 35% of your final grade.

A Note on Incompletes: I have a very strict, personal policy on incomplete grades: I will not give an incomplete grade unless the student has been ill for a good part of the semester and can obtain a doctor's medical certificate to authenticate the illness. NO EXCEPTIONS!

Your final grade will probably be based on the following percentages:

A: 90+ B: 80-90
C: 70-80 D: 60-70
F: < 60

About this course: There is a **lot** of material to cover in this course! I strongly suggest that students who take this course devote a couple of hours to the material each and every evening. If this is not possible, I would advise you strongly not to take this course. This is a rigorous introduction into mathematical analysis and advanced calculus. Sometimes, the course is of a computational nature but mostly the course deals with concepts, not numbers, and students are expected to be able to prove results based on material learned in the classroom. I will try to give a complete and understandable set of notes each lecture period, based on the material in the textbook. However, I strongly advise that students read the textbook since, quite often, there is an important perspective on some material that I will not have time to cover in class. Moreover, I strongly advise students to consult other textbooks by regularly visiting the library. If a student is serious about studying mathematics, one of the most important courses to master is beginning analysis and advanced calculus. Indeed, for a student to be successful at the graduate level, a sound understanding of the material in this course is absolutely essential.

Topics to be Covered in Math 5210-5220

1. **Review of Basic Concepts**: functions, algebra of sets, mathematical induction, truth tables, methods of proof, elementary logic, basic set theory, finite sets, countable sets, uncountable sets, bijections, equivalent sets, Schroeder-Bernstein theorem. (Chapter 1)

2. **Topology of $\mathbf{R}^n$**: open sets, closed sets, compactness, Bolzano-Weierstrass theorem, Heine-Borel theorem, connected sets, open sets, closure, boundary, interior. These concepts will also form the building blocks of future courses in analysis and topology. (Chapter 2)

3. **Convergence**: most students have already studied sequences in Calculus II. We review most of what you learned earlier but from a more rigorous point of view and study some new concepts. Topics to be covered include subsequences, monotone convergence theorem, Cauchy sequences, convergence of functions, uniform convergence. (Chapter 3)

4. **Continuous Functions**: again, most students have seen (or should have seen) the basics of continuity in calculus - this is the so-called ϵ-δ definition. We will spend quite a bit of time on this but we will also combine the ideas learned from the second topic (on Topology) above to re-look at the definition and properties of continuity. Topics to be covered include global continuity theorem, preservation of compactness, preservation of connectedness, uniform continuity, fixed point theorems, sequences of continuous functions, Bernstein and Weierstrass Approximation theorems. (Chapter 4)

5. **Functions of One Variable**: Students in Calculus I and II learned the basics of calculus; here we go indepth into this theory. Topics to include: differentiability of one-variable functions, mean value theorem and its consequences. (Chapter 5)

6. **Riemann-Stieltjes Integration**: Mathematics students have certainly studied the Riemann integral in Calculus courses and have seen many of its properties, although perhaps not very rigourously. We go indepth into this theory and, at the same time, study the more general Riemann-Stieltjes integral. Time permitting, we will also study an even more general integral (even more general than the Lebesgue integral which most students will study in a graduate course in analysis) called the generalized Riemann integral or, as its sometimes called, the Henstock-Kurzweil integral or gauge integral. (Chapter 5)

7. **Infinite Series**: In Calculus II, students learned the basics of infinite series as well as several tests for determining if an infinite series converges or not. In this part, we 're-do' these results but, again, more rigorously. We also consider a few more tests that are not usually covered in a beginning calculus sequence. Topics include Cauchy criterion, absolute convergence, Riemann's 'derangement' theorem, comparison test, limit comparison test, root test, ratio test, Raabe's test, integral test, Abel's partial summation formula, Dirichlet's test, alternating series test, series of functions, Weierstrass M-test, differentiation and integration of series term-by-term. Time permitting, we may delve a little bit into Fourier series. (Chapter 6)

8. **Differentiation in $\mathbf{R}^n$**: topics include partial derivatives, differentials, directional derivatives, gradient, chain rule, inverse and implicit function theorems, surfaces and tangents, derivatives of functions from $\mathbf{R}^n$ to $\mathbf{R}^m$, Jacobians, Mean Value Theorem, extremum problems, Lagrange multipliers. (Chapter 7 and supplementary notes)

9. **Integration in $\mathbf{R}^n$ and Vector Field Theory**: iterated integrals, multiple integrals, change of variables, Green's theorem in the plane, conservative vector fields, surface integrals, Stokes' Theorem, Divergence Theorem. (Chapter 8 and supplementary notes)

Recommended Additional Textbooks

1. *An Introduction to Analysis*, by William R. Wade; Prentice-Hall publishers.

2. *Elementary Analysis: The theory of calculus*, by Kenneth A. Ross; Springer-Verlag publishers.

3. *Principles of Mathematical Analysis,* by Walter Rudin, Mc-Graw-Hill publishers.

4. *A First Course in Real Analysis*, by M. H. Protter and C. B. Morrey, Springer-Verlag publishers.

5. *Elementary Classical Analysis*, by Jerrold E. Marsden, W. H. Freeman and Company publishers.

6. *Basic Elements of Real Analysis*, by M. H. Protter, Springer-Verlag publishers.

7. *Introduction to Real Analysis*, by John D. DePree and Charles W. Swartz, John Wiley & Sons publisher.

8. *Introduction to Real Analysis*, 3rd edition, by R. G. Bartle and D. R. Sherbert, John Wiley & Sons publisher.

9. *How to Prove It*, by Daniel J. Velleman, Cambridge University Press.

10. *How to Solve It,* by George Polya, Princeton University Press.

11. *Calculus of Vector Functions*, 3rd edition, by R. E. Williamson, R. H. Crowell, and H. F. Trotter, Prentice-Hall publishers.

12. *Advanced Calculus*, 4th edition, by Wilfred Kaplan, Addison-Wesley publishers.

13. *Advanced Calculus*, 3rd edition, by R. C. Buck, McGraw-Hill publishers.