Math 1210 Quiz #6
October 2, 2009

Name: Solutions
ID: A 00000000

Directions: Work all of the problems by the end of class, staple your homework to the back of the quiz. Make sure your name is on the top of this paper.

Problem 1. (7 p.238) Find $\frac{dy}{dx}$ using implicit differentiation.

$$x^2y^2 + xsin(y) = 4$$

$$\frac{d}{dx}(x^2y^2 + x\sin(y)) = \frac{d}{dx}(4)$$

$$\Rightarrow 2xy^2 + x^2\left(2y\frac{dy}{dx}\right) + \sin(y) + x\cos(y)\frac{dy}{dx} = 0$$

$$\Rightarrow (2xy^2 + \sin(y)) + (2x^2y + x\cos(y))\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2xy^2 + \sin(y))}{2x^2y + x\cos(y)}$$

Problem 2. (15 p.239) Find the equation of the tangent line to the ellipse defined by the implicit relationhip

$$x^2 + xy + y^2 = 3$$

at the point $(1, 1)$.

$$\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(3)$$

$$\Rightarrow 2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow (2x + y) + (x + 2y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$$

For $x = 1, y = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2+1)}{1+2} = -1$$

So,

$$(y-1) = (-1)(x-1)$$

is the appropriate tangent line

Problem 3. (12 p.245) Compute the derivative of the following function.

$F(y) = y\ ln\,(1 + e^y)$

$$\frac{dF}{dy} = \left(\frac{d}{dy}(y)\right)\ln(1+e^y) + y\,\frac{d}{dy}\ln(1+e^y)$$

$$= \ln(1+e^y) + y\cdot\frac{1}{1+e^y}\cdot\frac{d}{dy}(1+e^y)$$

$$= \ln(1+e^y) + \frac{y}{1+e^y}(0+e^y)$$

$$= \ln(1+e^y) + \frac{ye^y}{1+e^y}$$

Problem 4. (16 p.245) Compute the derivative of the following function.

$G(u) = ln\sqrt{\dfrac{3u+2}{3u-2}}$

Hint: Using properties of logarithms may make the differentiation easier.

For this problem, start with

$$G(u) = \ln\sqrt{\frac{3u+2}{3u-2}}$$

$$= \ln\left(\frac{\sqrt{3u+2}}{\sqrt{3u-2}}\right) = \ln\left(\sqrt{3u+2}\right) - \ln\left(\sqrt{3u-2}\right)$$

$$= \frac{1}{2}\ln(3u+2) - \frac{1}{2}\ln(3u-2)$$

$$G'(u) = \frac{1}{2}\cdot\frac{1}{3u+2}\cdot\frac{d}{du}(3u+2) - \frac{1}{2}\,\frac{1}{3u-2}\cdot\frac{d}{du}(3u-2)$$

$$= \frac{3}{2}\left(\frac{1}{3u+2} - \frac{1}{3u-2}\right)$$

or if they stop here $\longrightarrow$

$$= \frac{3}{2}\left(\frac{\cancel{3u}-2-\cancel{3u}-2}{9u^2-4}\right)$$

$$= \frac{3}{2}\left(\frac{-4}{9u^2-4}\right) = 6\cdot\frac{1}{4-9u^2}$$

Math 1210 HW #6 Solutions

Section 3.5 7, 10, 15, 18, 19, 26, 32, 36

#7. $F(x) = \sqrt[4]{1+2x+x^2} = (1+2x+x^2)^{1/4}$

$$\frac{dF}{dx} = \frac{1}{4}(1+2x+x^2)^{-3/4} \cdot \frac{d}{dx}(1+2x+x^2)$$

$$= \frac{1}{4}(1+2x+x^2)^{-3/4}(2+2x)$$

$$= \frac{1}{2}\frac{1+2x}{(1+2x+x^2)^{3/4}}$$

#10. $f(t) = \sqrt[3]{1+\tan(t)} = (1+\tan(t))^{1/3}$

$$\frac{df}{dt} = \frac{1}{3}(1+\tan(t))^{-2/3} \cdot \frac{d}{dt}(1+\tan(t))$$

$$= \frac{1}{3}(1+\tan(t))^{-2/3}(0+\sec^2(t))$$

$$= \frac{\sec^2(t)}{3(1+\tan(t))^{2/3}}$$

#15 $y = xe^{-x^2}$

$$\frac{dy}{dx} = \left(\frac{d}{dx}(x)\right)e^{-x^2} + x \cdot \frac{d}{dx}e^{-x^2}$$

$$= (1)e^{-x^2} + x\, e^{-x^2} \cdot \frac{d}{dx}(-x^2)$$

$$= e^{-x^2} + x\, e^{-x^2}(-2x)$$

$$= e^{-x^2}(1-2x^2)$$

#18. $h(t) = (t^4-1)^3(t^3+1)^4$

$$h'(t) = \left(\frac{d}{dt}(t^4-1)^3\right)(t^3+1)^4 + (t^4-1)^3\left(\frac{d}{dt}(t^3+1)^4\right)$$

$$= \left(3(t^4-1)^2 \cdot \frac{d}{dt}(t^4-1)\right)(t^3+1)^4 + (t^4+1)^3 \cdot (4)(t^3+1)^3 \cdot \frac{d}{dt}(t^3+1)$$

$$= \left(3(t^4-1)^2(4t^3)\right)(t^3+1)^4 + 4(t^4-1)^3(t^3+1)^3(3t^2)$$

ok if you get to here

$$= 12(t^4-1)^2(t^3+1)^3\left(t^3(t^3+1) + (t^4-1)t^2\right)$$

$$= 12(t^4-1)^2(t^3+1)^3\left(t^6+t^3+t^6-t^2\right)$$

$$= 12(t^4-1)^2(t^3+1)^3\left(2t^6+t^3-t^2\right)$$

#19.

$$y = e^{x\cos(x)}$$

$$y' = e^{x\cos(x)} \cdot \frac{d}{dx}(x\cos(x))$$

$$= e^{x\cos(x)} \cdot (\cos(x) + x(-\sin(x)))$$

$$= e^{x\cos(x)} \cdot (\cos(x) - x\sin(x))$$

#26.

$$y = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

$$y' = \frac{\left(\frac{d}{du}(e^u - e^{-u})\right)(e^u + e^{-u}) - (e^u - e^{-u})\left(\frac{d}{du}(e^u + e^{-u})\right)}{(e^u + e^{-u})^2}$$

$$= \frac{(e^u - e^{-u}(-1))(e^u + e^{-u}) - (e^u - e^{-u})(e^u + e^{-u}(-1))}{(e^u + e^{-u})^2}$$

$$= \frac{(e^u + e^{-u})^2 - (e^u - e^{-u})^2}{(e^u + e^{-u})^2}$$

ok to here

$$= \frac{\cancel{e^{2u}} + 2e^u e^{-u} + \cancel{e^{-2u}} - \cancel{e^{2u}} + 2e^u e^{-u} - \cancel{e^{-2u}}}{(e^u + e^{-u})^2}$$

$$= \frac{4}{(e^u + e^{-u})^2}$$

This is $\boxed{\text{sech}(u)}$

#32. If

$$y = \sqrt{x+\sqrt{x+\sqrt{x}}}$$
$$= \left(x + \left(x + x^{1/2}\right)^{1/2}\right)^{1/2}$$

then

$$y' = \frac{1}{2}\left(x+\left(x+x^{1/2}\right)^{1/2}\right)^{-1/2} \cdot \frac{d}{dx}\left(x + \left(x+\sqrt{x}\right)^{1/2}\right)$$
$$= \frac{1}{2}\left(x+\left(x+x^{1/2}\right)^{1/2}\right)^{-1/2} \cdot \left(1 + \frac{1}{2}\left(x+\sqrt{x}\right)^{-1/2} \cdot \frac{d}{dx}\left(x+\sqrt{x}\right)\right)$$
$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{1}{\left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \left(1+\frac{1}{2\sqrt{x}}\right)\right)}$$

#36 For $(x,y) = (0,0)$ and

$$y = \sin(x) + \sin^2(x)$$

we can write

$$y - y_0 = y'(x_0)\,(x - x_0)$$

when

$$y_0 = 0$$
$$y' = \cos(x) + 2\sin(x)\cos(x)$$
$$\Rightarrow y'(0) = \cos(0) + 2\sin(0)\cos(0)$$
$$= 1 + 0 = 1$$

$$x_0 = 0$$

$$\Rightarrow y - 0 = (1)(x-0)$$
$$\Rightarrow y = x$$

Section 3.6 5, 6, 8, 10, 12, 16, 18, 29, 30

#5. $x^2y + xy^2 = 3x$

$\Rightarrow \frac{d}{dx}(x^2y + xy^2) = \frac{d}{dx}(3x)$

$\Rightarrow 2xy + x^2\frac{dy}{dx} + y^2 + x\left(2y\frac{dy}{dx}\right) = 3$

$\Rightarrow 2xy + y^2 + (x^2 + 2xy)\frac{dy}{dx} = 3$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{3 - 2xy - y^2}{x^2 + 2xy}}$

#6. $y^5 + x^2y^3 = 1 + ye^{x^2}$

$\Rightarrow 5y^4\frac{dy}{dx} + 2x\cdot y^3 + x^2\left(3y^2\frac{dy}{dx}\right) = 0 + \frac{dy}{dx}e^{x^2} + y\left(e^{x^2}\cdot(2x)\right)$

$\Rightarrow \left(5y^4 + 3x^2y^2 - e^{x^2}\right)\frac{dy}{dx} = 2xye^{x^2} - 2xy^3$

$\Rightarrow \frac{dy}{dx} = \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}$

#8. If

$1 + x = \sin(xy^2)$

then

$\frac{d}{dx}(1+x) = \frac{d}{dx}\left(\sin(xy^2)\right)$

$\Rightarrow 0 + 1 = \cos(xy^2)\cdot\frac{d}{dx}(xy^2)$

$\Rightarrow 1 = \cos(xy^2)\left(y^2 + x(2y)\frac{dy}{dx}\right)$

$\Rightarrow 1 = y^2\cos(xy^2) + 2xy\cos(xy^2)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}$

10. $y\sin(x^2) = x\sin(y^2)$

$$\Rightarrow \frac{d}{dx}\left(y\sin(x^2)\right) = \frac{d}{dx}\left(x\sin(y^2)\right)$$

$$\Rightarrow \frac{dy}{dx}\sin(x^2) + y\cos(x^2)\cdot(2x) = \sin(y^2) + x\cos(y^2)\left(2y\frac{dy}{dx}\right)$$

$$\Rightarrow \left(\sin(x^2) - 2xy\cos(y^2)\right)\frac{dy}{dx} = \sin(y^2) - 2xy\cos(x^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin(y^2) - 2xy\cos(x^2)}{\sin(x^2) - 2xy\cos(y^2)}$$

#12. $\sin(x) + \cos(y) = \sin(x)\cos(y)$

$$\frac{d}{dx}\left(\sin(x) + \cos(y)\right) = \frac{d}{dx}\left(\sin(x)\cos(y)\right)$$

$$\Rightarrow \cos(x) - \sin(y)\frac{dy}{dx} = \cos(x)\cos(y) - \sin(x)\sin(y)\cdot\frac{dy}{dx}$$

$$\Rightarrow \left(\sin(x)\sin(y) - \sin(y)\right)\frac{dy}{dx} = \cos(x)\cos(y) - \cos(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(x)(\cos(y) - 1)}{\sin(y)(\sin(x) - 1)}$$

#16. For

$$x^2 + 2xy - y^2 + x = 2$$

and $(x_0, y_0) = (1, 2)$ we can compute

$$2x + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} + 1 = 0 \qquad \Rightarrow \boxed{y - 2 = \tfrac{7}{2}(x-1)}$$

and

$$2(1) + 2(2) + 2(1)\frac{dy}{dx} - 2(2)\frac{dy}{dx} + 1 = 0$$

$$\Rightarrow 7 - 2\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = 7/2$$

#18 We have

$$x^{2/3}+y^{2/3}=4 \Rightarrow \frac{d}{dx}\left(x^{2/3}+y^{2/3}\right)=0$$

$$\Rightarrow \tfrac{2}{3}x^{-1/3}+\tfrac{2}{3}y^{-1/3}\cdot\frac{dy}{dx}=0$$

For $x=-3\sqrt{3}$, $y=1$

$$\Rightarrow \frac{1}{(-3\sqrt{3})^{1/3}}+\frac{1}{1^{1/3}}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{3}}$$

So,

$$y-1=\frac{1}{\sqrt{3}}\left(x-(-3\sqrt{3})\right)$$

or

$$y-1=\frac{1}{\sqrt{3}}\left(x+3\sqrt{3}\right)$$

#29 $y=\tan^{-1}\sqrt{x}$

$$\frac{dy}{dx}=\frac{1}{1+(\sqrt{x})^2}\cdot\frac{d}{dx}\sqrt{x}$$

$$=\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}(1+x)}$$

#30. $y=\sqrt{\tan^{-1}x}$

$$\frac{dy}{dx}=\frac{1}{2}\left(\tan^{-1}(x)\right)^{-1/2}\cdot\frac{1}{1+x^2}$$

$$=\frac{1}{2\sqrt{\tan^{-1}(x)}\,(1+x^2)}$$

Section 3.7 4, 6, 7, 8, 13, 16, 19, 20, 30, 32, 34

#4. $f(x) = \cos(\ln(x))$

$$\Rightarrow f'(x) = -\sin(\ln(x)) \cdot \frac{d}{dx}\ln(x)$$
$$= -\sin(\ln(x)) \cdot \frac{1}{x}$$
$$= \frac{-\sin(\ln(x))}{x}$$

#6. $f(x) = \ln\sqrt[5]{x} = \frac{1}{5}\ln(x)$

$$\Rightarrow f'(x) = \frac{1}{5} \cdot \frac{d}{dx}\ln(x)$$
$$= \frac{1}{5} \cdot \frac{1}{x} = \frac{1}{5x}$$

#7. $f(x) = \log_2(1-3x)$

$$\Rightarrow f'(x) = \frac{1}{1-3x} \cdot \frac{1}{\ln(2)} \cdot \frac{d}{dx}(1-3x)$$
$$= \frac{-3}{\ln(2)(1-3x)}$$

#8. $f(x) = \log_5(xe^x)$

$$= \log_5(x) + \log_5 e^x$$

$$f'(x) = \frac{1}{x} \cdot \frac{1}{\ln(5)} + \frac{1}{\cancel{e^x}} \cdot \frac{1}{\ln(5)} \cdot \cancel{e^x}$$
$$= \frac{1}{x}\,\frac{1}{\ln(5)} + \frac{1}{\ln(5)}$$
$$= \frac{1}{\ln(5)}\left(\frac{1}{x} + 1\right)$$

#13. $f(u) = \dfrac{\ln(u)}{1+\ln(2u)} = \dfrac{\ln(u)}{1+\ln(2)+\ln(u)}$

$$f'(u) = \frac{\frac{1}{u}\left(1+\ln(2)+\cancel{\ln(u)}\right) - \cancel{\ln(u)\cdot\left(0+0+\tfrac{1}{u}\right)}}{\left(1+\ln(2)+\ln(u)\right)^2}$$

$$= \frac{\frac{1}{u}\left(1+\ln(2)\right)}{\left(1+\ln(2u)\right)^2} = \frac{1+\ln(2)}{u\left(1+\ln(2u)\right)^2}$$

#16. $G(u) = \ln\sqrt{\dfrac{3u+2}{3u-2}}$

See solution on quiz

#19. $y = e^x \ln(x)$

$$y' = e^x \ln(x) + e^x \frac{d}{dx}\ln(x)$$

$$= e^x \ln(x) + \frac{1}{x}e^x$$

$$y'' = e^x \ln(x) + e^x \cdot \frac{1}{x} - \frac{1}{x^2}e^x + \frac{1}{x}e^x$$

$$= e^x\left(\ln(x) + \frac{2}{x} - \frac{1}{x^2}\right)$$

#20. $y = \dfrac{\ln(x)}{x^2}$

$$y' = \frac{\left(\frac{1}{x}\right)\cdot x^2 - \ln(x)\cdot(2x)}{x^4}$$

$$= \frac{x - 2x\ln(x)}{x^4} = \frac{1-2\ln(x)}{x^3}$$

$$y'' = \frac{\left(0-\frac{2}{x}\right)x^3 - \left(1-2\ln(x)\right)(3x^2)}{x^6} = \frac{-2x^2 - \left(1-2\ln(x)\right)3x^2}{x^6}$$

$$= \frac{-2-3\left(1-2\ln(x)\right)}{x^4}$$

30 Start with

$$\ln(y) = \frac{1}{4} \ln\left(\frac{x^2+1}{x^2-1}\right)$$

$$= \frac{1}{4}\ln(x^2+1) - \frac{1}{4}\ln(x^2-1)$$

$$\Rightarrow \frac{d}{dx}\ln(y) = \frac{1}{y}\frac{dy}{dx} = \frac{1}{4}\cdot\frac{1}{x^2+1}\cdot(2x) - \frac{1}{4}\frac{1}{x^2-1}\cdot(2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{x}{x^2+1} - \frac{x}{x^2-1}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt[4]{\frac{x^2+1}{x^2-1}}\left(\frac{x(x^2-1) - x(x^2+1)}{(x^2+1)(x^2-1)}\right)$$

$$\vdots$$

32

$$y = x^{\cos(x)}$$

$$\Rightarrow \ln(y) = \cos(x)\ln(x)$$

$$\Rightarrow \frac{1}{y}\frac{d}{dx}(y) = -\sin(x)\ln(x) + \cos(x)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = x^{\cos(x)}\cdot\left(\frac{\cos(x)}{x} - \sin(x)\cos(x)\right)$$

#34

$$y = \sqrt{x}^{\,x}$$

$$\Rightarrow \ln y = x \ln x^{1/2}$$

$$= \frac{x}{2}\ln x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\ln(x) + \frac{x}{2}\cdot\frac{1}{x}$$

$$= \frac{1}{2}(\ln(x) + 1)$$

$$\Rightarrow \frac{dy}{dx} = (\sqrt{x})^x\cdot\left(\frac{1}{2}\right)(\ln(x)+1)$$