Problem Definition

Problem 29. Evaluate the definite integral.

\[ \int_0^1 x^2 e^x \, dx \]

Solution Step 1:

For this problem, we can apply the following definitions for integration by parts.

\[
\begin{align*}
    u &= x^2 & du &= 2x \, dx \\
    dv &= e^x \, dx & v &= e^x
\end{align*}
\]

The application of these definitions produces

\[
\int_0^1 x^2 e^x \, dx = \left( x^2 e^x \right)_0^1 - \int_0^1 2x e^x \, dx
\]

\[
= (1^2 e^1) - (0^2 e^0) - \int_0^1 2x e^x \, dx
\]

\[
= e - 2 \int_0^1 x e^x \, dx
\]

Solution Step 2:

To complete the work on this problem, we still need to do an additional integration by parts. For this step we can use

\[
\begin{align*}
    u &= x & du &= dx \\
    dv &= e^x \, dx & v &= e^x
\end{align*}
\]

The application of these definitions produces

\[
\begin{align*}
    e - 2 \int_0^1 x e^x \, dx &= e - 2 \left( x e^x |_0^1 - \int_0^1 e^x \, dx \right) \\
    &= e - 2 \left( (1) e^1 - (0) e^0 \right) - e^x |_0^1 \\
    &= e - 2 \left( (1) e^1 - (0) e^0 \right) \\
    &= e - 2 \left( e - e^1 \right) \\
    &= e - 2 \left( e - 1 \right) \\
    &= e - 2e + 2e - 2 \\
    &= e - 2 \approx 0.718
\end{align*}
\]