Problem Definition

Problem 37. Evaluate the definite integral.

\[ \int_{1}^{3} \frac{e^{\frac{3}{x}}}{x^2} \, dx \]

Solution Step 1:

Let’s start by computing the antiderivative

\[ \int \frac{e^{\frac{3}{x}}}{x^2} \, dx \]

using the substitution \( u = \frac{3}{x} \) which requires that

\[ du = -\frac{3}{x^2} \, dx \]

The indefinite integral is computed using

\[
\int \frac{e^{\frac{3}{x}}}{x^2} \, dx = \int \frac{1}{\left(3\right)} e^{\frac{3}{x}} \cdot \frac{3}{x^2} \, dx = -\frac{1}{3} \int e^u \, du = -\frac{1}{3} e^u + C
\]

Transforming back to the original variable the antiderivative is

\[ \int \frac{e^{\frac{3}{x}}}{x^2} \, dx = -\frac{1}{3} e^{\frac{3}{x}} + C \]

Solution Step 2:

Now, we need to compute the definite integral. Since any antiderivative will do we can choose \( C = 0 \) in the antiderivative we computed in the previous
step. The value of the definite integral is

\[
\int_1^3 \frac{e^{3/x}}{x^2} \, dx = \left( -\frac{1}{3} e^{3/x} \right) \bigg|_1^3 \\
= \left( -\frac{1}{3} e^{3/(3)} \right) \\
- \left( -\frac{1}{3} e^{3/(1)} \right) \\
= \left( -\frac{1}{3} e^1 + \frac{1}{3} e^3 \right) \\
= \frac{1}{3} e^1 (e^2 - 1) \\
= \approx 5.789
\]