Problem Definition

Problem 45. Use any basic integration formulas to compute the following indefinite integral.

$$\int 4e^{2x-1} \, dx$$

Solution Step 1:

The problem requires that we perform a simple substitution. We start with

$$u = 2x - 1$$

The idea of this simple substitution is to transform to a simpler argument in the natural exponential. The substitution requires $du = 2dx$. The integral can be rewritten as follows.

$$\int 4e^{2x-1} \, dx = \int 2e^{2x-1}(2dx)$$

$$= \int 2e^u du$$

$$= 2 \int e^u du$$

Solution Step 2:

The indefinite can be computed using the following.

$$2 \int e^u du = 2e^u + C$$

where $C$ is the constant of integration.

Solution Step 3:

The last step in the process is to get the antiderivative back in terms of the original variables. This means

$$\int 4e^{2x-1} \, dx = 2e^u + C = 2e^{2x-1} + C$$
Solution Step 4:

The result can be checked by differentiating our answer to make sure that the antiderivative is correct. This is done as follows.

\[
\frac{d}{dx} \left( 2e^{2x-1} + C \right) = \frac{d}{dx} \left( 2e^{2x-1} \right) + \frac{d}{dx} (C)
\]

\[
= 2 \frac{d}{dx} \left( e^{2x-1} \right) + (0)
\]

\[
= 2e^{2x-1} \frac{d}{dx} (2x - 1)
\]

\[
= 2e^{2x-1} (2)
\]

\[
= 4e^{2x-1}
\]