Problem Definition

Problem 53. **Demand Function** Find the demand function \( x = f(p) \) that satisfies the initial conditions.

\[
\frac{dx}{dp} = -\frac{6000p}{(p^2 - 16)^{3/2}}
\]

When \( x = 5000 \) we have \( p = \$5 \).

**Solution Step 1:**

To start, we need to compute the general antiderivative for the given rate of change of demand with respect to price. The integral we need to compute is

\[
\int \frac{dx}{dp} \, dp = \int \left( -\frac{6000p}{(p^2 - 16)^{3/2}} \right) \, dp
\]

To do this we will need to do a substitution. The idea is to substitute for the trouble spot in the integral which is the denominator. In particular, the expression in the power. So, we can define

\( u = p^2 - 16 \)

The substitution requires \( du = 2p \, dp \). We can rewrite the integral as follows.

\[
\int \left( -\frac{6000p}{(p^2 - 16)^{3/2}} \right) \, dp = \int \left( -3000 \frac{2p}{(p^2 - 16)^{3/2}} \right) \, dp
\]

\[
= -3000 \int \left( \frac{1}{(p^2 - 16)^{3/2}} \right) (2p \, dp)
\]

\[
= -3000 \int \left( \frac{1}{u^{3/2}} \right) \, du
\]

\[
= -3000 \int u^{-3/2} \, du
\]

This is a much easier integral to work with.

**Solution Step 2:**
The general antiderivative is computed as follows.

\[-3000 \int u^{-3/2} du = -3000 \left( \frac{1}{-1/2} u^{-1/2} \right) + C = 6000u^{-1/2} + C\]

**Solution Step 3:**

For this problem we will want the antiderivative in terms of the original variables. This is done as follows.

\[x = \int \frac{dx}{dp} = 6000u^{-1/2} + C = 6000(p^2 - 16)^{-1/2} + C\]

**Solution Step 1:**

The last step in the process is to enforce the condition on the price and demand values given in the problem. When price is \( p = 5 \), the demand, \( x = 5000 \), should be predicted by the model. So,

\[5000 = 6000(5^2 - 16)^{-1/2} + C = 6000 \frac{1}{3} + C = 2000 + C\]

To make this work, \( C = 3000 \) and the model requires

\[x = 6000(p^2 - 16)^{-1/2} + 3000 = \frac{6000}{\sqrt{p^2 - 16}} + 3000\]