Problem Definition
Problem 39. Find the indefinite integral and check your result by differentiation.

\[ \int \frac{1}{3x^4} \, dx \]

Solution Step 1:

To make the problem a bit easier we can rewrite the integrand to use the power rule. That is,

\[ \frac{1}{3x^4} = \frac{1}{3} x^{-4} \]

Solution Step 2:

The power rule will allow us to compute this indefinite integral. The rule is

\[ \int x^n \, dx = \frac{1}{n+1} x^n + C_1 \]

where \( C_1 \) is the constant of integration that must be included.

For the present problem \( n = -3 \), so

\[
\int \frac{1}{3x^4} \, dx = \int \frac{1}{3} x^{-4} \, dx \\
= \frac{1}{3} \int x^{-4} \, dx \\
= \frac{1}{3} \left( \frac{1}{-3} \right) x^{-3} + C \\
= -\frac{1}{9} x^{-3} + C \\
= -\frac{1}{9x^3} + C
\]

Solution Step 3:
Now, let’s check the answer. We differentiate our result above with respect to $x$.

$$\frac{d}{dx} \left( -\frac{1}{9} x^{-3} + C \right) = \frac{d}{dx} \left( \frac{1}{9} x^{-3} \right) + \frac{d}{dx} (C)$$

$$= -\frac{1}{9} \frac{d}{dx} (x^{-3}) + (0)$$

$$= -\frac{1}{9} (-3) (x^{-4})$$

$$= \frac{1}{3} (x^{-4})$$

$$= \frac{1}{3x^4}$$