Problem Definition

Problem 45. **Demand:** The demand function for a product is modeled by

\[ p = 5000 \left( 1 - \frac{4}{4 + e^{-0.002x}} \right) \]

Find the price of the product if the quantity demanded is (a) \( x = 100 \) units and (b) \( x = 500 \) units. What is the limit of the price as \( x \) increases without bound.

**Solution Step 1:**

For the two demand values, we need to evaluate the expression above. For the first value (a) \( x = 100 \) we can write

\[ p = 5000 \left( 1 - \frac{4}{4 + e^{-0.002(100)}} \right) = 5000 \left( 1 - \frac{4}{4 + e^{-0.2}} \right) \approx 849.53 \]

and for the second value

\[ p = 5000 \left( 1 - \frac{4}{4 + e^{-0.002(500)}} \right) = 5000 \left( 1 - \frac{4}{4 + e^{-1.0}} \right) \approx 421.12 \]

The value seems to be decreasing.

**Solution Step 2:**

To answer the question about the value as the demand increases, we can take the limit as \( x \to \infty \). That is, see what the horizontal limit would be. This is computed using information from Section 3.6.

\[ \lim_{x \to \infty} 5000 \left( 1 - \frac{4}{4 + e^{-0.002x}} \right) = 5000 \lim_{x \to \infty} \left( 1 - \frac{4}{4 + e^{-0.002x}} \right) \]

Since the function is continuous, as \( x \to \infty \)

\[ \lim_{x \to \infty} e^{-0.002x} = 0 \]

This means that

\[ 5000 \lim_{x \to \infty} \left( 1 - \frac{4}{4 + e^{-0.002x}} \right) = 5000 \lim_{x \to \infty} \left( 1 - \frac{4}{4 + 0} \right) = 5000 \lim_{x \to \infty} (1 - 1) = 5000 \lim_{x \to \infty} 0 = 0 \]

So, the price will go to zero as the demand gets arbitrarily large.