Problem Definition

Problem 11. **Cost, Revenue, and Profit** A company that produces sports supplements calculates that it costs and revenue can be modeled by the equations

\[ C = 125,000 + 0.75x \quad R = 250x - \frac{1}{10}x^2 \]

where \( x \) is the number of units of sport supplements produced in a week. If production in one particular week is 1000 units and is increasing at a rate of 150 units per week, find:

(a) the rate at which the cost is changing,
(b) the rate at which the revenue is changing, and
(x) the rate at which the profit is changing.

**Solution Step 1:**

For part (a) we need to compute the derivative of both sides of the cost equation with respect to time, \( t \). This gives

\[ \frac{dC}{dt} = 0.75 \frac{dx}{dt} \]

Using the rate of change in production gives

\[ \frac{dC}{dt} = 0.75(150) = 112.25 \]

dollars per week.

**Solution Step 2:**

For part (b) we need to compute the derivative of both sides of the cost equation with respect to time, \( t \). This gives

\[ \frac{dR}{dt} = 250 \frac{dx}{dt} - \frac{1}{5} x \frac{dx}{dt} \]

Using the rate of change in production gives

\[ \frac{dR}{dt} = 250(150) - \frac{1}{5}(1000)(150) = 7500 \]
dollars per week.

Solution Step 3:

For part (c) we know the profit is the revenue minus the cost. So, we can write

\[ \frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} \]

So, we can use the results from parts (a) and (b)

\[ \frac{dP}{dt} = 7500 - 112.5 = 7387.5 \]

dollars per week.