Problem Definition

Problem 23. Find the given value.

\[ f(x) = \sqrt{4 - x} \quad f'''(-5) \]

Solution Step 1:

The first step is to compute the first, second, and third derivatives to get to the appropriate derivative for evaluation. The needed derivatives come from the following calculations

\[ f'(x) = \frac{d}{dx} \sqrt{4 - x} = \frac{d}{dx} (4 - x)^{\frac{1}{2}} \]
\[ = \frac{1}{2} (4 - x)^{-\frac{1}{2}} (-1) \]
\[ = -\frac{1}{2} (4 - x)^{-\frac{1}{2}} \]

\[ f''(x) = \frac{d}{dx} \left( -\frac{1}{2} (4 - x)^{-\frac{1}{2}} \right) \]
\[ = \frac{1}{4} (4 - x)^{-\frac{3}{2}} (-1) \]
\[ = -\frac{1}{4} (4 - x)^{-\frac{3}{2}} \]

\[ f'''(x) = \frac{d}{dx} \left( -\frac{1}{4} (4 - x)^{-\frac{3}{2}} \right) \]
\[ = \frac{3}{8} (4 - x)^{-\frac{5}{2}} (-1) \]
\[ = -\frac{3}{8} (4 - x)^{-\frac{5}{2}} \]

Solution Step 2:

With the third derivative available we can evaluate this function at

\[ f'''(-5) = -\frac{3}{8} (4 - (-5))^{-\frac{5}{2}} = -\frac{3}{8} (9)^{-\frac{5}{2}} = -\frac{1}{648} \]