Problem Definition

Problem 53. Find the limit if it exists.

\[ \lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x} \]

Solution Step 1:

Since this is a rational function of polynomials, the first step is to evaluate each of the limits of the polynomials separately. The limit for the numerator is

\[ \lim_{\Delta x \to 0} (2(x + \Delta x) - 2x) = 0 \]

meaning the numerator is approaching zero as \( x \) gets close to \(-2\). The limit for the denominator is the following

\[ \lim_{\Delta x \to 0} \Delta x = 0 \]

Solution Step 2:

If the limit in the denominator is zero, we need to see what the limit in the numerator is. Since this limit is also zero we cannot determine a value at this point. Instead we need to do a bit of algebra to proceed.

\[ 2(x + \Delta x) - 2x = 2x + 2\Delta x - 2x = \Delta x \]

Since a limit only considers points near \( \Delta x = 0 \) we can write

\[ \lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 2 \]

where the factors of \( \Delta x \) has been cancelled.

Solution Step 3: Putting things together gives

\[ \lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x} = \lim_{\Delta x \to 0} 2 = 2 \]

In this case the limit exists, is equal to 2, and can be found after a bit of algebra.