Problem Definition

Problem 35. Determine whether the function

\[ y = \frac{4}{x} \]

is a solution for the following differential equation

\[ y^{(4)} - 16y = 0 \]

Solution Step 1:

The idea is to substitute the given function \( y(t) \) and the fourth derivative of \( y(t) \) into the equation to see if the equation is true for this choice. So, for

\[ y = \frac{4}{x} = 4x^{-1} \]

we compute the derivative with respect to \( t \). This is

\[
\begin{align*}
y' &= -4x^{-2} \\
y'' &= 8x^{-3} \\
y''' &= -24x^{-4} \\
y^{(4)} &= -96x^{-5}
\end{align*}
\]

Solution Step 2:

Now we try out the function and fourth derivative. For the ode

\[ y^{(4)} - 16y = 0 \]

Substituting the function and the fourth derivative into the left hand side we obtain

\[ y^{(4)} - 16y = -96x^{-5} - 16 \left( 4x^{-1} \right) = -96x^{-5} - 64x^{-1} \neq 0 \]

Since the left hand side cannot be zero for this function, the given function cannot be a solution of the ordinary differential equation.