Moab Topology Conference 2012: Abstracts

Expository talks

Helen Wong (Carleton College), An Introduction to the Kauffman Skein Algebra

The Jones polynomial of a link has a well-known and particularly elegant description using the Kauffman bracket. In generalizing it, one is led quickly to the Kauffman bracket for links in a thickened surface $S \times [0, 1]$, which can be endowed with the structure of an algebra. We will explore the many interpretations of the resulting Kauffman skein algebra, using the language of combinatorial skein theory, of the Witten-Reshetikhin-Turaev topological quantum field theory, and, most excitingly, of hyperbolic geometry.

Ilya Kofman (CUNY), Knots, graphs and surfaces

Many important knot invariants have been discovered using associated graphs and surfaces. In this introductory talk, we highlight some recent applications of ribbon graphs and state surfaces to polynomial invariants, categorification, and the topology of the knot complement.

Roland Van der Veen (UC Berkeley), The colored Jones polynomial

In this talk we will review various approaches to the Jones polynomial and the web of conjectures it stars in. In particular we attempt a gentle, non-technical sketch of Witten's original path integral and how it beautifully motivates conjectures such as the volume conjecture.

Research talks

Ian Agol (UC Berkeley), The virtual Haken conjecture

We prove that cubulated hyperbolic groups are virtually special. The proof relies on results of Haglund and Wise which also imply that they are linear groups, and quasi-convex subgroups are separable. A consequence is that closed hyperbolic 3-manifolds have finite-sheeted Haken covers, which resolves the virtual Haken question of Waldhausen and Thurston's virtual fibering question. Part of the result is joint work with Groves and Manning.

Cody Armond (Louisiana State University), Adequate links and the head and tail of the colored Jones polynomial

The colored Jones polynomial is a link invariant, which produces a sequence of Laurent polynomials. Viewing the individual coefficients as sequence, we can show that for adequate links coefficients stabilize. This allows us to define power two power series called the head and tail of the colored Jones polynomial. For many knots they can be computed and turn out to be interesting number-theoretic q-series. We also show that for any adequate link, there is an alternating knot with the same head and tail.

Oliver Dasbach (Louisiana State University), Properties of the colored Jones polynomial of a knot and the relations to its geometry

The colored Jones polynomial is one of the more mysterious invariants in knot theory. Starting with the classical Jones polynomial the colored Jones polynomial is a sequence of Laurent polynomials, indexed by some color N.

We will discuss properties of certain coefficients of the colored Jones polynomial that persist for increasing N. Furthermore, we will give some relations to the geometry of the knot complement.

This is ongoing work with Cody Armond.

Allison Gilmore (Princeton), Knot Floer homology and $gl(1|1)$ quantum invariants

Knot Floer homology is a homology theory for knots whose graded Euler characteristic is the Alexander polynomial. Although it originally arose from Heegaard Floer homology, Ozsvath and Szabo later developed an algebraic cube of resolutions (relying on a Floer–theoretic invariant of singular knots) that computes knot Floer homology without reference to Heegaard diagrams or holomorphic disks. We show that a straightforward generalization of their construction produces an invariant of framed graphs. We then consider the decategorification of this invariant in relation to Viro's $gl(1|1)$ quantum relative of the Alexander polynomial. The result is an understanding of the Ozsvath–Szabo cube of resolutions as a categorification of the quantum topology construction of the Alexander polynomial.

Neil Hoffman (Boston College), Meridians of hyperbolic knot complements

Given a finite volume hyperbolic 3–orbifold $Q = H^3/\Gamma$, one can ask if Q is covered by a knot complement $S^3 - K$. Using the standard definition that a meridian is a curve c such that Dehn filling along c yields S^3. Considering a Wirtinger presentation for a knot group, we see that a knot group is generated by a set of conjugates of a meridian. The 6 Theorem of Agol and Lackenby places restrictions on the possible meridians a knot group in Γ. Along similar lines, number theoretic information carried by a sufficiently nice representation of Γ into $PSL(2, \mathbb{C})$ can be used to restrict the possible meridians of knot groups that are subgroups of Γ. I will discuss what these number theoretic restrictions are and apply these techniques to show that the dodecahedral knot complements are the only knot complements in their commensurability class.

Ilya Kofman (CUNY), Lorenz and horseshoe knots

The Lorenz flow is the prototypical chaotic dynamical system with a "strange attractor". Lorenz knots are periodic orbits in the Lorenz flow on $\mathbb{R}^3$. Horseshoe knots are periodic orbits in the flow on $\mathbb{R}^3$ given by the suspension of Smale's horseshoe map. In this talk, I will provide some background, and discuss some surprising relationships between these knots and the simplest hyperbolic knots.

Heather Russell (University of Southern California), Odd cohomology of type A Springer varieties

Springer varieties are of interest in geometric representation theory because their cohomology carries an action of the symmetric group. In fact, the top degree cohomology is always an irreducible representation. Springer varieties are also related to Khovanov's categorification of the Jones polynomial. A new categorified knot invariant called odd Khovanov homology suggests the existence of a parallel odd theory of Springer varieties. In this talk we will review a simple algebraic description of the cohomology of Springer varieties. We will then describe joint work with Aaron Lauda defining the odd cohomology of Springer varieties.

Abigail Thompson (UC Davis), Distance 2 Heegaard splittings

It is well–known that the distance of a Heegaard splitting is related to global properties of the underlying 3–manifold. For example, Casson–Gordon show that existence of a splitting of distance one implies that the manifold is Haken. Distance zero splittings are either stabilized, or the underlying manifold is reducible. We study the case of distance two splittings. These are interesting as many known examples of splittings of distance greater than one in fact have distance precisely two. Our aim is to describe how simple conditions on a distance two splitting imply interesting properties of the underlying manifold. Joint with J.H. Rubinstein.

Anastasiia Tsvietkova (University of Tennessee, Knoxville), An alternative approach to hyperbolic structures on link complements

Thurston demonstrated that every link in a 3–sphere is a torus link, a satellite link or a hyperbolic link and these three categories are mutually exclusive. It also follows from work of Menasco that an alternating link represented by a prime diagram is either hyperbolic or a $(2, n)$–torus link.

A new method for computing the hyperbolic structure of the complement of a hyperbolic link, based on ideal polygons bounding the regions of a diagram of the link rather than decomposition of the complement into ideal tetrahedra, was suggested by M. Thistlethwaite. Although the method is applicable to all diagrams of hyperbolic links under a few mild restrictions, it works particularly well for alternating (non-torus) links. The talk will introduce the basics of the method. Some applications will be discussed, including a surprising rigidity property of certain tangles, and formulas that allow one to calculate hyperbolic and complex volume of 2–bridged links directly from the diagram.

Roland Van der Veen (UC Berkeley), The slope conjecture

The slope conjecture gives a topological interpretation for the degree of the colored Jones polynomial in terms of surfaces in the knot complement. We will show how the conjecture is motivated by the AJ conjecture and present some new results including its behaviour under twisting and a generalisation for links.

Helen Wong (Carleton College), Representations of the Kauffman Skein Algebra

The Kauffman skein algebra not only lies at the core of quantum topology, but bears deep meaning in hyperbolic geometry. We will describe briefly how to construct representations of the Kauffman skein algebra using the quantum Teichmuller space. We will also describe invariants, based on the Chebyshev polynomials, to tell the representations apart. This is joint work with Francis Bonahon, and we will address some of the many “miraculous cancellations” that appear in our proofs of the above.