Stat 1040, Normal Approximation

1. Given the box [1 , 1 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0] , draw 100 times with replacement and consider the sum of the draws.

a) What is the EV for the sum of the draws?

$= \text{Box AV} \times \text{number of draws} = \frac{2}{10} \times 100$

$= 20$

b) Find the SE for the sum of the draws.

$= \text{Box SD} \times \sqrt{n} = \sqrt{\frac{1}{5} \cdot \frac{4}{5}} \times \sqrt{100}$

$= \frac{2}{5} \times 10 = 4$

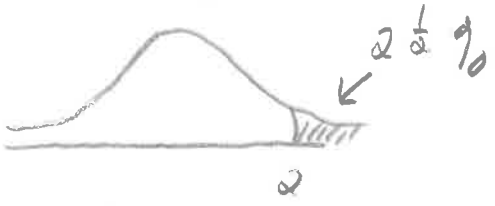
c) Find the probability that the sum of the draws is between 16 and 22. What is the chance that it is greater than 28 ?

The sum of the draws follows the normal curve.

$\frac{16-20}{4} = -1$, $\frac{22-20}{4} = .5$

$\frac{A(1)}{2} + \frac{A(.5)}{2} = 53\%$

$\frac{28-20}{4} = 2$



2. Current registration in a local voting district shows that 80 percent of the voters are Republican with 20 percent Democrat. If you randomly sample (with replacement) 100 voters, find the chance of getting more than 28 Democrats in your sample.

Box Model is in problem 1.

3. Toss a coin 100 times. Find the probability of getting more than 60 heads.

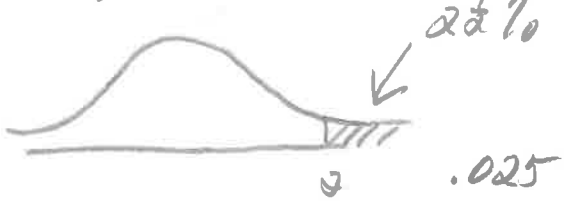
[0, 1] → Draw 100 times & consider the sum of draws

The sum of draws follows the normal curve.

$EV = \frac{1}{2} \times 100 = 50$

$SE = \sqrt{\frac{1}{2} \cdot \frac{1}{2}} \times \sqrt{100} = 5$

$\frac{60-50}{5} = 2$



4. A die is rolled 180 times.

a) The total number of spots should be around _____, give or take _____ or so.

[1, 2, 3, 4, 5, 6] → Draw 180 & consider sum

$EV = (3.5)(180) = 630$

$SE = \text{Box SD} \times \sqrt{180}$

$= (1.7)\sqrt{180} = 22.8 \approx 23$

b) The number of aces should be around 630, give or take SE or so.

630 ± 23

c) Find the chance of getting less than 40 aces.

[1, 0, 0, 0, 0, 0] → Draw 180 times & consider sum

$EV = \frac{1}{6} \cdot 180 = 30$

$SE = \sqrt{\frac{1}{6} \cdot \frac{5}{6}} \times \sqrt{180} = 5$

$\frac{40-30}{5} = 2$

$A(2) = 95\%$

$2\frac{1}{2}\%$

15

Sampling without replacement from a LARGE population is just like sampling with replacement.

Example: A large crop of apples has an average weight of 4.3 oz with an SD of 1.5 oz. You choose 100 apples at random. What's the chance the total weight is less than 25 pounds?

Box AV = 4.3 oz

Box SD = 1.5 oz.

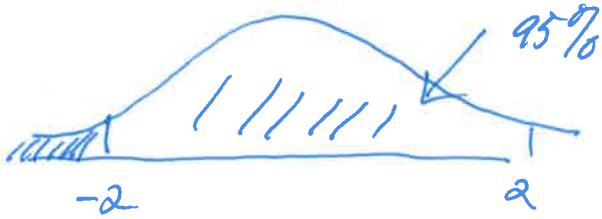
Draw 100 with replacement. Consider the sum of draws.

EV for sum = $(4.3)100 = 430$

SE for sum = $(1.5)\sqrt{100} = 15$

16 x 25 = 400

$$\frac{400 - 430}{15} = -2$$



$2\frac{1}{2}\%$

Example: Suppose 10% of people in a large population are "underweight". If we take a random sample of 1000 people from this population, what is the chance that more than 103 will be "underweight"?

[1, 0, 0, 0, 0, 0, 0, 0, 0, 0] → Draw 1000 with replacement + consider the sum of the draws.

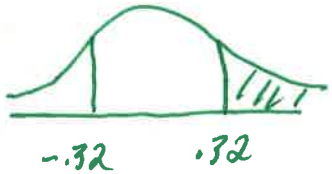
"follows the normal curve"

Box AV = $\frac{1}{10}$

Box SD = $\sqrt{\frac{1}{10} \cdot \frac{9}{10}} = \frac{3}{10}$

EV for sum = $\frac{1}{10} \cdot 1000 = 100$, SE for sum = $\frac{3}{10} \cdot \sqrt{1000} \approx 9.48$

$$\frac{103 - 100}{9.48} = \approx .32$$



A(.32) = 24%

$\frac{76}{2} = 38\%$