Table 1. John Kerrich's coin-tossing experiment. The first column shows the number of tosses. The second shows the number of heads. The third shows the difference

$$\text{number of heads} - \text{half the number of tosses.}$$

Number of tosses	*Number of heads*	*Difference*	*Number of tosses*	*Number of heads*	*Difference*
10	4	−1	600	312	12
20	10	0	700	368	18
30	17	2	800	413	13
40	21	1	900	458	8
50	25	0	1,000	502	2
60	29	−1	2,000	1,013	13
70	32	−3	3,000	1,510	10
80	35	−5	4,000	2,029	29
90	40	−5	5,000	2,533	33
100	44	−6	6,000	3,009	9
200	98	−2	7,000	3,516	16
300	146	−4	8,000	4,034	34
400	199	−1	9,000	4,538	38
500	255	5	10,000	5,067	67

Figure 1. Kerrich's coin-tossing experiment. The "chance error" is

number of heads − half the number of tosses.

This difference is plotted against the number of tosses. As the number of tosses goes up, the size of the chance error tends to go up. The horizontal axis is not to scale and the curve is drawn by linear interpolation.

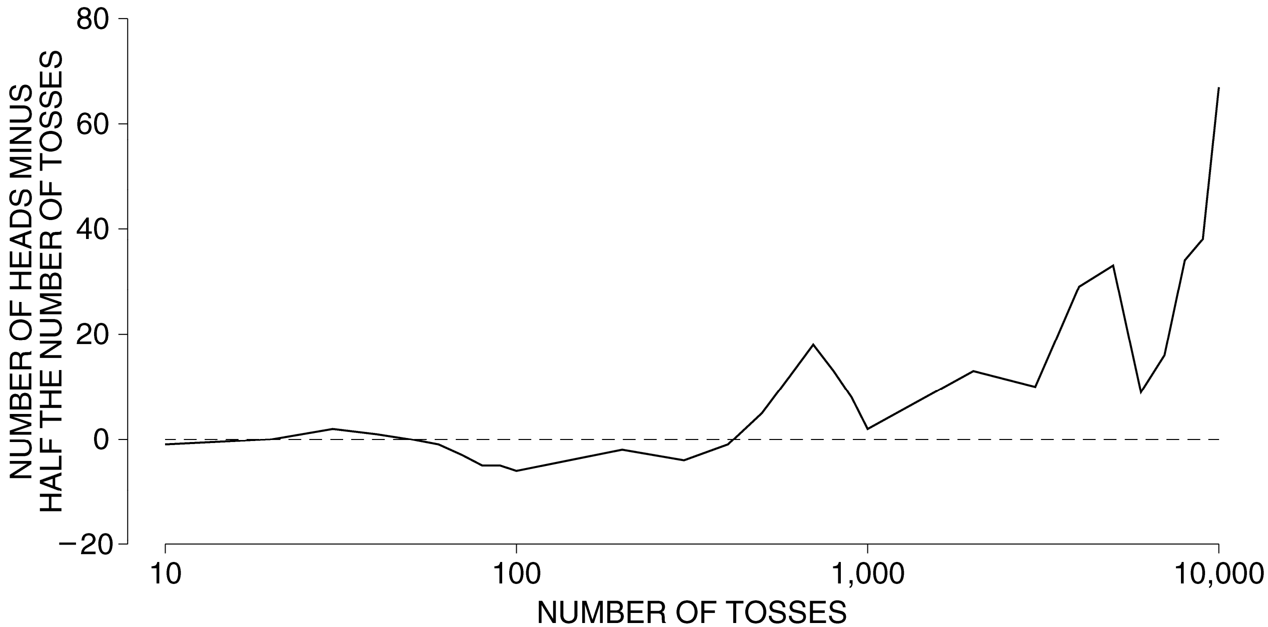
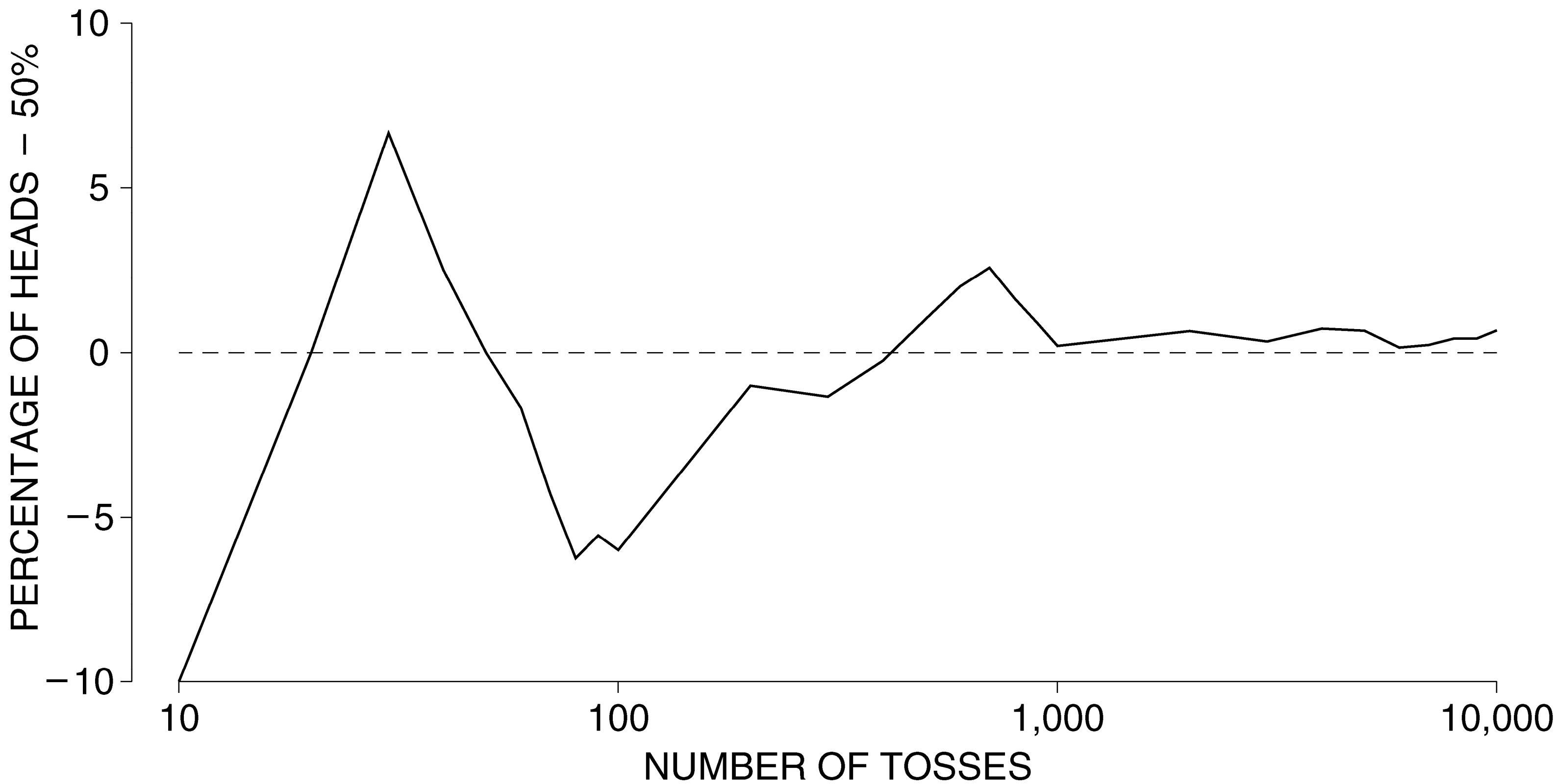


Figure 2. The chance error expressed as a percentage of the number of tosses. When the number of tosses goes up, this percentage goes down: the chance error gets smaller relative to the number of tosses. The horizontal axis is not to scale and the curve is drawn by linear interpolation.



- **Chapter 16: The Law of Averages**

If we toss a coin many times,

number of H’s = half the number of tosses + **chance error**

The “law of averages” says that for a large number of tosses, the **chance error** is likely to be

- LARGE in absolute terms
- SMALL compared to the number of tosses

In fact, as the number of tosses increases, the **chance error** is likely to get

- LARGER in absolute terms
- SMALLER compared to the number of tosses

In terms of percentages, the “law of averages” says that as the number of tosses increases, the **percentage of H’s** is likely to get closer and closer to 50%, but it is less and less likely to be exactly 50%.

In fact, as the number of tosses increases, the **percentage error** is likely to get smaller and smaller.

- **Example 1.** A coin will be tossed and you win $1 if the number of heads is exactly equal to the number of tails. Which is better for you, 10 tosses or 1000?

- **Example 2.** A coin will be tossed and you win $1 if the percentage of heads is between 40% and 60%. Which is better for you, 100 tosses or 1000?

- **Example 3.** You are betting on tosses of a coin: if the coin lands heads, you win $1, if it lands tails, you lose $1. The last 10 tosses have all been heads. What's the chance the next toss is a head?

- **Example 4.** You play a game in which you win \$1 if the percentage of heads is 60% or more. Which is better for you, 100 tosses or 1000 tosses?

- **Example 5.** You play a game in which a die is rolled and you win $1 if the percentage of "6"s is 20% or more. Which is better for you, 100 rolls or 1000 rolls?

- **Chance Processes**

- Chance processes are ones that are affected by chance error.

- Examples:
- number of H’s when tossing a coin
- amount of money won when playing a game of chance
- percentage of Democrats in a random sample of people

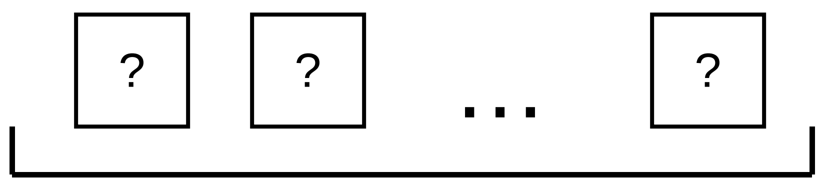
- Box models help us to answer the question:
- “ how large is the chance error likely to be?”

- **Box Models**

- Box models make an analogy between a chance process and drawing tickets from a box. Usually, the analogy goes like this:

- **The *quantity of interest* is like the ____?____ of ___?___ draws from the box:**

-



- **Box Models**

- To make a box model, answer the following questions:

- What is the quantity of interest? Are we interested in
- the sum of the draws
- the average of the draws?
- the percentage of 1’s in the draws?

- How many draws?

- How many tickets go in the box?

- What numbers go on the tickets?

- **Example 6.** You play a game in which you roll a die 10 times and get paid the amount shown on the die (each time). Find a box model for the **total amount you win**.

- **Example 7.** You play a game in which you roll a die 10 times. Each time a “6” occurs, you win \$10, otherwise you lose \$1. Find a box model for the **total amount you win**.

- **Example 8.** A multiple-choice test has 20 questions, each with 4 possible choices. Each correct answer is worth 5 points, and for each incorrect answer you lose 2 points. Find a box model for **your test score** if you <u>guess</u> all the answers.