Chapter 14: More about Chance

Listing the Ways

Sometimes you can figure chances by listing all the ways an event can happen and counting or adding up the individual chances.

Example 1.

Roll 2 dice. Find the chance the total number of spots is 7.

6 and 1

5 and 2

3 and 4

?????????

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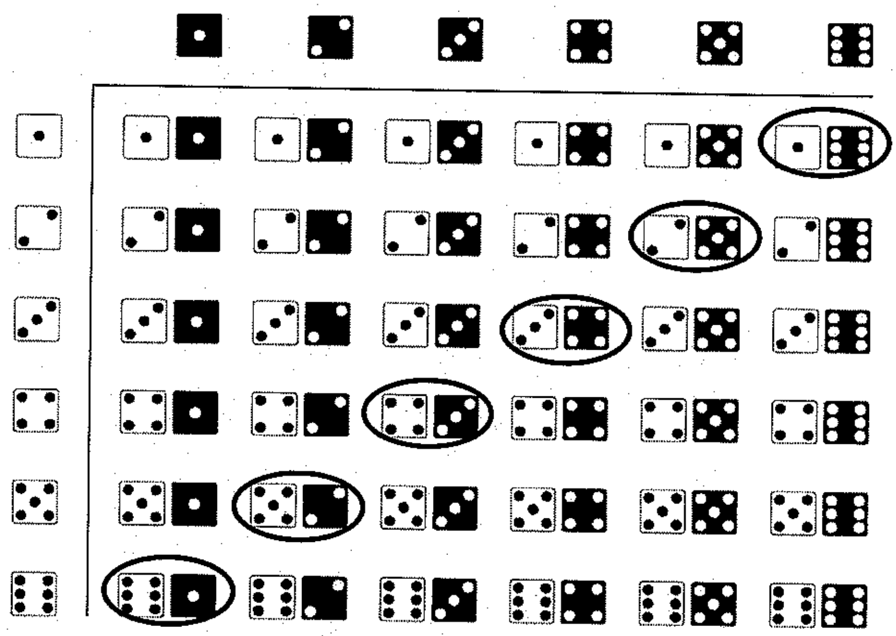


Each of the 36 possibilities has chance 1/36. The chance of rolling a "7" is 6/36 or 1/6.

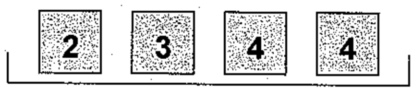
Example 2. Draw 2 tickets at random with replacement from the box: 2 3 4 4

a) What is the chance that the tickets show the same number?

b) What is the chance the first number is smaller than the second number?

c) What is the chance that the sum of the numbers is odd?

Example 3. Draw 2 tickets at random without replacement from the box:



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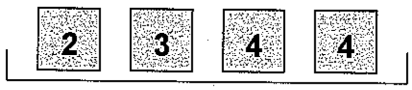

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Gamblers in 17th century Italy bet on the total number of spots that showed up when rolling 3 dice. They reasoned as follows:

The chance of getting 9 spots is the same as the chance of getting 10 spots because

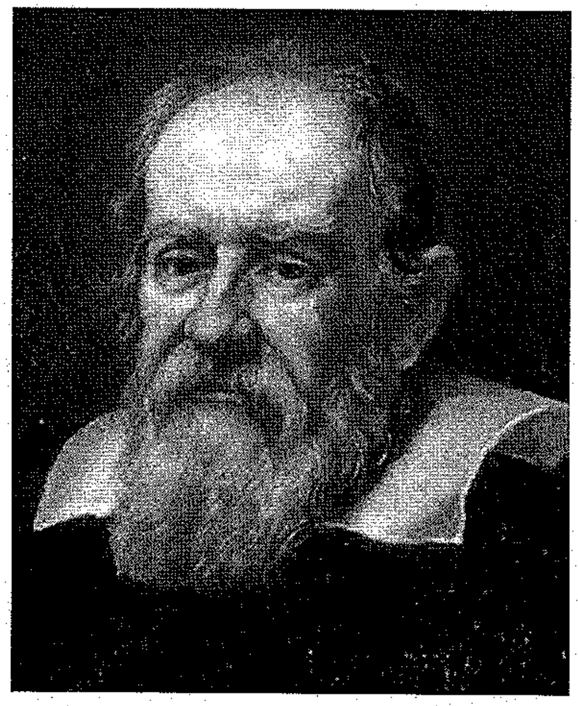
the ways of getting "9" are:

1 2 6   1 3 5   1 4 4   2 3 4   2 2 5   3 3 3

the ways of getting "10" are:

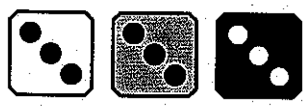
1 4 5   1 3 6   2 2 6   2 3 5   2 4 4   3 3 4

BUT they knew from experience that "10" was more likely than "9" so they called in Galileo to help. Can you see their mistake?

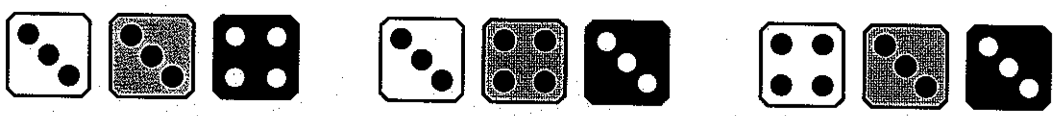


Galileo (Italy, 1564 – 1642)

Galileo: There is only one way to get 3 3 3:



and 3 ways to get 3 3 4:



Triplets for 9	*Number of ways to roll each triplet*
1 2 6	6
1 3 5	6
1 4 4	3
2 3 4	6
2 2 5	3
3 3 3	1
Total	25

Triplets for 10	*Number of ways to roll each triplet*
1 4 5	6
1 3 6	6
2 2 6	3
2 3 5	6
2 4 4	3
3 3 4	3
Total	27

BY MARILYN VOS SAVANT

Ask Marilyn

I will never read your column again.
—Eric Gatterdam, Tucson, Ariz.

I have lost nearly all my faith in you.
—Douglas Kraft, Notre Dame, Ind.

I can only conclude that you are not woman enough to face the truth and admit your mistake. You are highly intelligent, and that is an admirable quality, but high intelligence coupled with an unwillingness to admit a mistake is unforgivable.
—Leonard Haefele, Overland Park, Kan.

It really puzzles and frustrates me that, despite your great perspicacity, you are unable to see that your answer to the "man and woman each with two children" problem is wrong.
—J.H. Wuller, St. Louis, Mo.

You are wrong. This is borne out by the application of Bayes' rule to the probability structure you imposed, and in the inner refinement functionality as given in the Dempster-Shafer theory of evidential reasoning.
—Dave Ferkinhoff, Middletown, R.I.

I was horrified to read that one of your few supporters was an engineer responsible for assessing risks in the operation of nuclear power plants. I sometimes wonder why critics of IQ testing don't point to some of your work as vivid examples of the vast difference between IQ and logic.
—Robert Williamson, Knoxville, Tenn.

As an anti-nuclear activist, I find it both scary and humorous that a person with a Ph.D. in nuclear engineering who once managed the performance of probabilistic safety analyses of nuclear power plants thinks you are correct.
—Ben Davis Jr., Sacramento, Calif.

I guess the real hope is that the nuclear engineer wasn't paying very close attention when she offered her assent, or else the next problem will involve three-eyed children.
—Jason Zeamon, White Bear Lake, Minn.

That question about a woman and a man, each with two children, is causing controversy again. But this time our women readers are asked to participate, and $1000 is on the line.

I am writing to the Nuclear Regulatory Commission to suggest that any power plants approved for operation by Jennifer Adams be closed immediately.
—Russell Redgate, Marstons Mills, Mass.

This is not going to go away until you admit that you are wrong, wrong, wrong!!!
—Pearl Meibos, Salt Lake City, Utah

You are not the only genius to base logic on a faulty major premise. Einstein did it more than once.
—Margaret-Mary del Tufo, North Myrtle Beach, S.C.

Even the Bulls lose one every once in a while.
—Chris Rowley, Frisco, Tex.

I will send $1000 to your favorite charity if you can prove me wrong. The chances of both the woman and the man having two boys are equal.
—Eldon Moritz, Arlington, Tex.

You're on, Eldon! If you are wrong, you'll donate $1000 to the American Heart Association. If I'm wrong, I'll donate $1000 to that association. Rather than explain my reasoning again, let's just put it to the test. Here's the original problem:

"A woman and a man (unrelated) each have two children. At least one of the woman's children is a boy, and the man's older child is a boy. Do the chances that the woman has two boys equal the chances that the man has two boys?"

I said the chances that the woman has two boys are 1 in 3 and the chances that the man has two boys are 1 in 2. The letter-writers agree with me about the man. But they disagree with me about the woman. Instead, they say the chances that the woman has two boys are 1 in 2 (just like the man's chances).

Readers, here's how you can help prove which answer about the woman is correct. To my women readers: If you have exactly two children (no more), and at least one of them is a boy (either child or both of them), write —or send e-mail—and tell me the sex of both of your children. Don't consider their ages.

In other words, it's fine to write if your older child is a boy and your younger child is a girl. It's also fine to write if your older child is a girl and your younger child is a boy. And it's fine to write if both of your children are boys. I need to hear from all of you (but only if you have two children and no more).

We'll publish the results in an upcoming column.

Mutually Exclusive

Two things are mutually exclusive if they cannot both happen (at the same time).

Examples:

- Toss a coin. Heads and Tails are mutually exclusive.
- Toss 2 coins. Each of the following are mutually exclusive: HH, HT, TH, TT.
- Roll a die. Getting a "6" and getting an odd number are mutually exclusive. Getting a "6" and an even number are NOT mutually exclusive, because "6" is even.
- In a family of 4 children, getting exactly 2 boys and getting exactly 3 boys are mutually exclusive.
- Toss 3 coins. Getting HHH and getting TTT are mutually exclusive.
- Toss 3 coins. Getting TTT and getting *at least one* H are mutually exclusive.
- Toss 3 coins. Getting HHH and getting *at least one* H are NOT mutually exclusive.

The Addition Rule

The chance that one thing OR another thing happens is the SUM of the individual chances, IF the things are mutually exclusive.

Examples:

- Draw a card from a pack of 52 cards:

  chance of ACE is 4/52

  chance of QUEEN is 4/52

  so the chance of ACE or QUEEN is 8/52

- Toss 3 coins:

  chance of TTH is 1/8

  chance of THT is 1/8

  chance of HTT is 1/8

  so the chance of exactly 1 H is 3/8

Example 4. Toss 2 coins.

a) Find the chance that you get at least one H.

b) Find the chance that the first coin lands H or the second coin lands H.

Example 5. Roll 2 dice.

a) Find the chance that you get 

b) Find the chance that you DON'T get 

Example 5. Roll 2 dice.

a) Find the chance that ***both*** of the dice show 

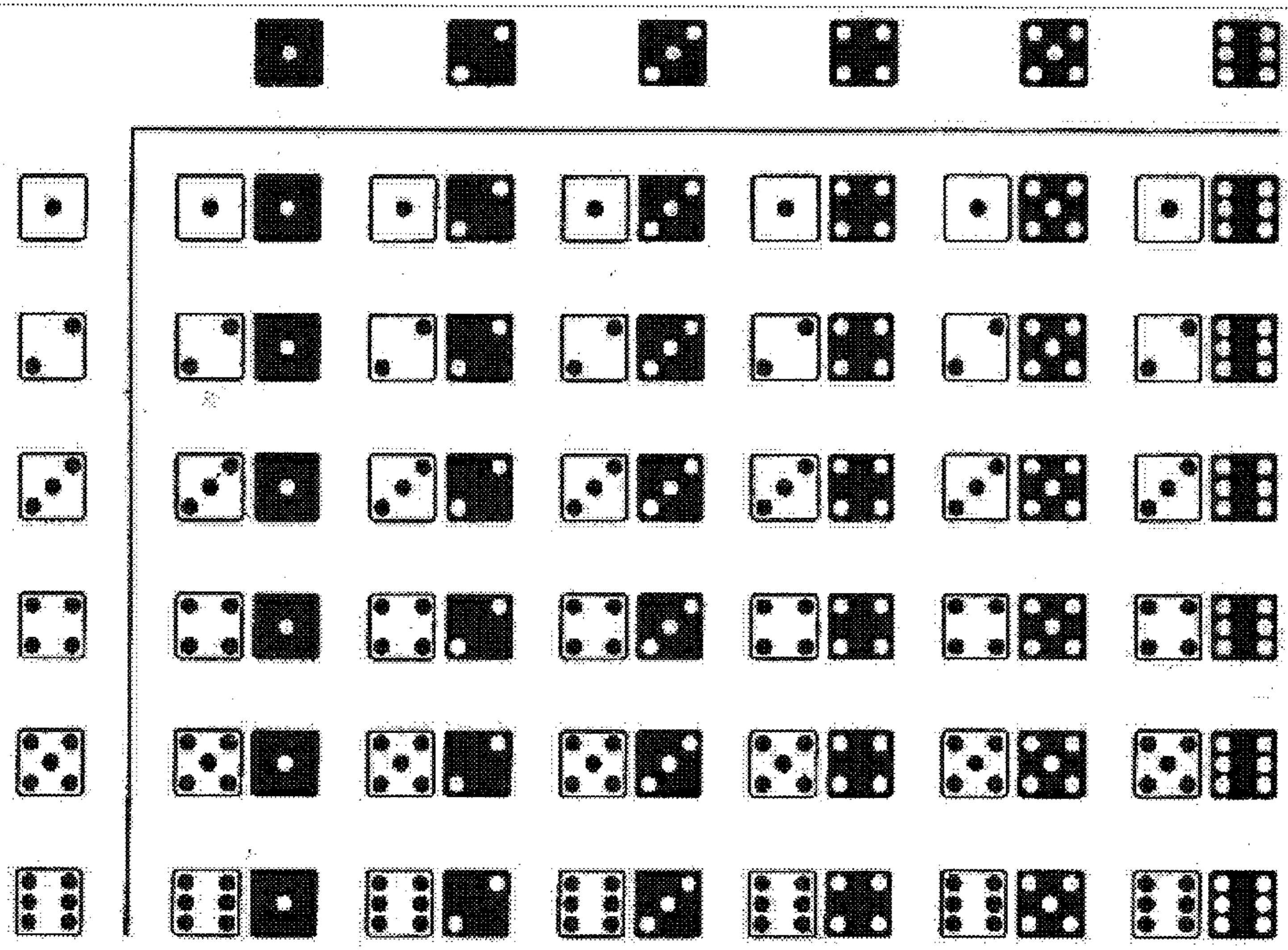
b) Find the chance that ***neither*** of the dice show 

c) Find the chance that the dice are ***not both*** 

d) Find the chance that ***at least one*** of the dice shows 

e) Find the chance that ***exactly one*** of the dice shows 

Dice : Sample Space

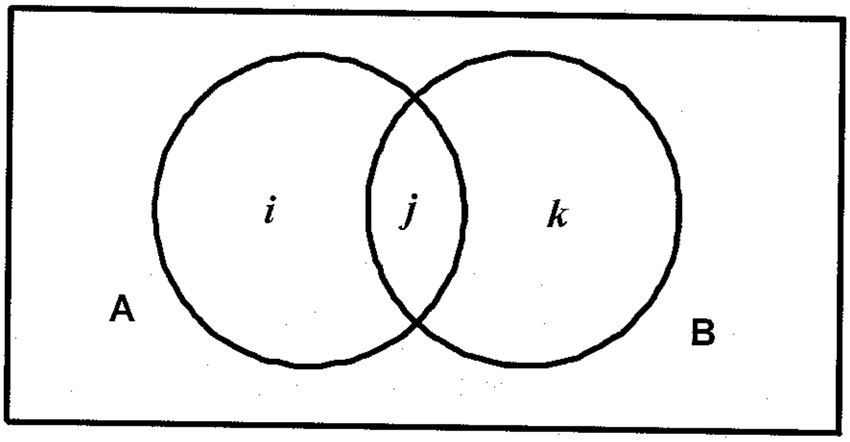


COIN TOSSING: SAMPLE SPACE, N = 4

0 - tails:	**HHHH**			
1 - tail:	**THHH**	**HTHH**	**HHTH**	**HHHT**
2 - tails:	**TTHH**	**THTH**	**THHT**	
	HTTH	**HTHT**	**HHTT**	
3 - tails:	**HTTT**	**THTT**	**TTHT**	**TTTH**
4 - tails:	**TTTT**			

GENERAL ADDITION RULE

Outcome Space



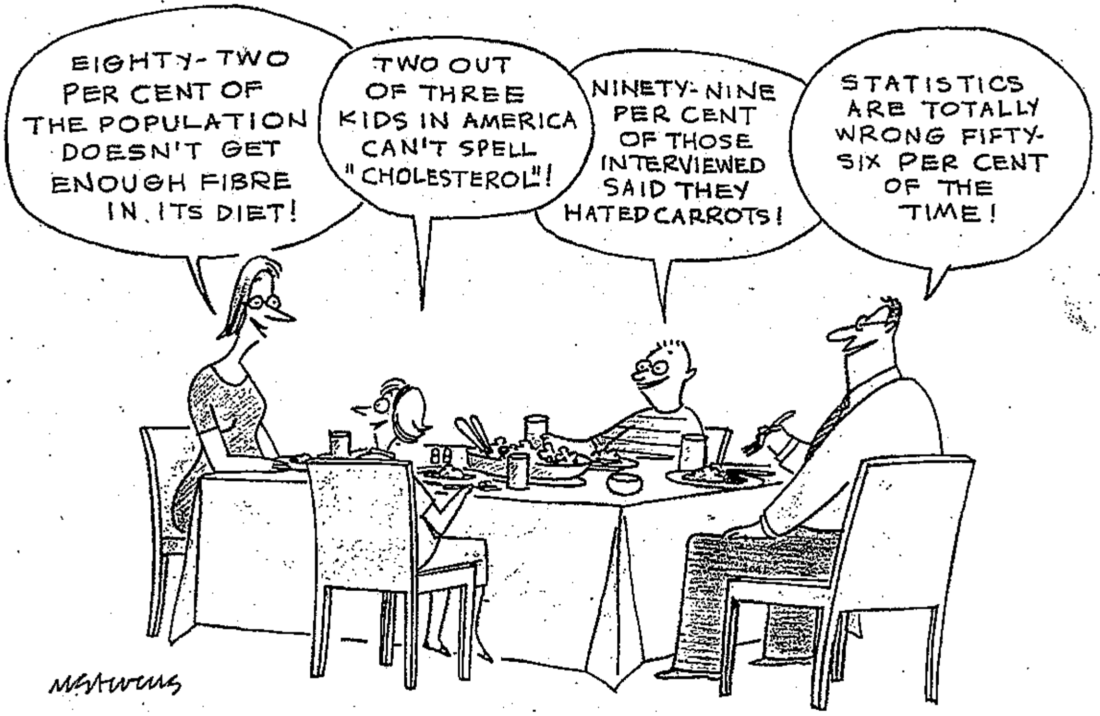
n equally likely outcomes

Probability of $A = \frac{i+j}{n}$ Probability of $B = \frac{j+k}{n}$

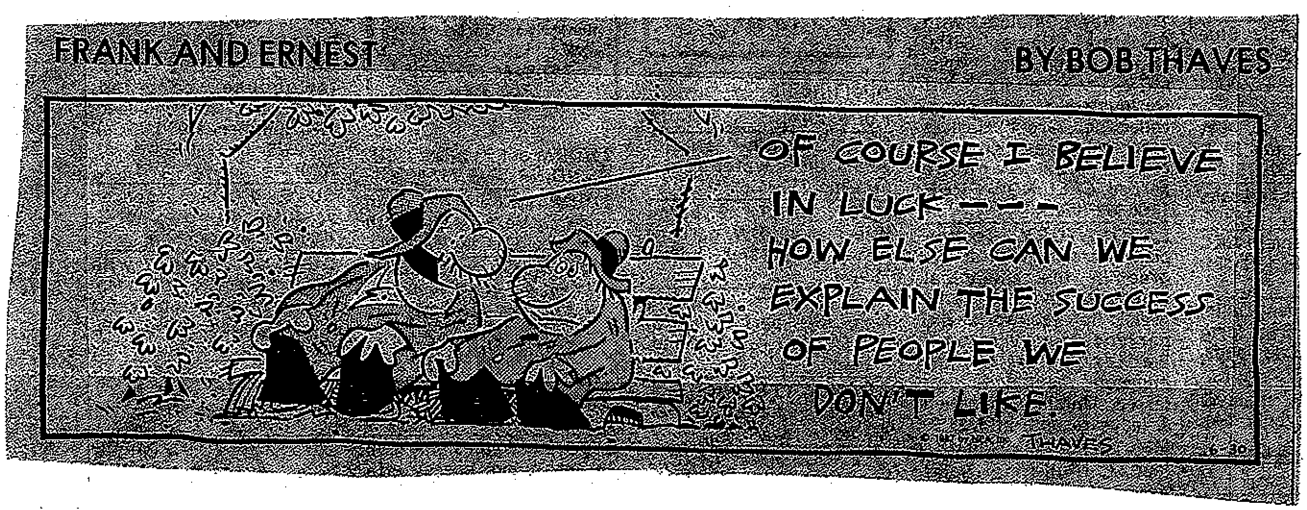
Probability of $A \text{ and } B = \frac{j}{n}$

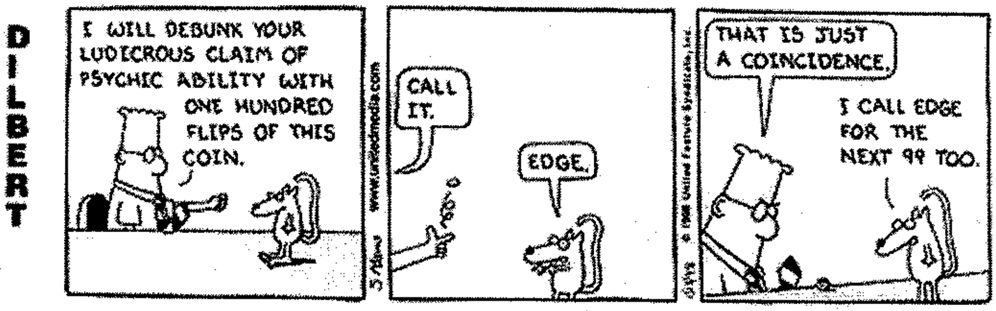
Probability of $A \text{ or } B = \frac{i+j+k}{n} = \frac{i+j}{n} + \frac{j+k}{n} - \frac{j}{n}$

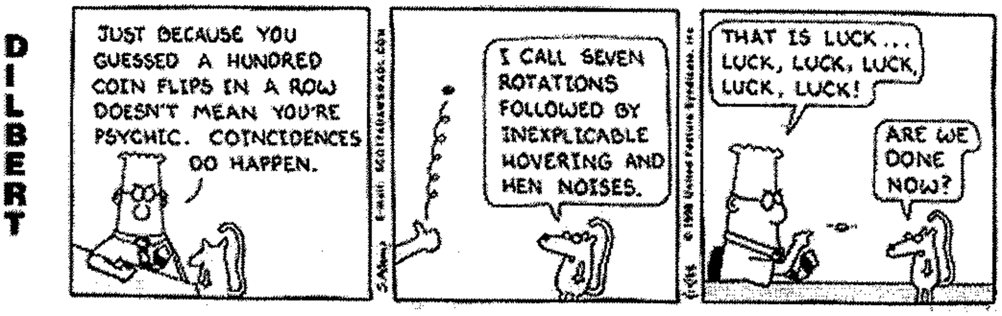
Probability of $A \text{ or } B$ = Probability of A + Probability of B − Probability of $A \text{ and } B$



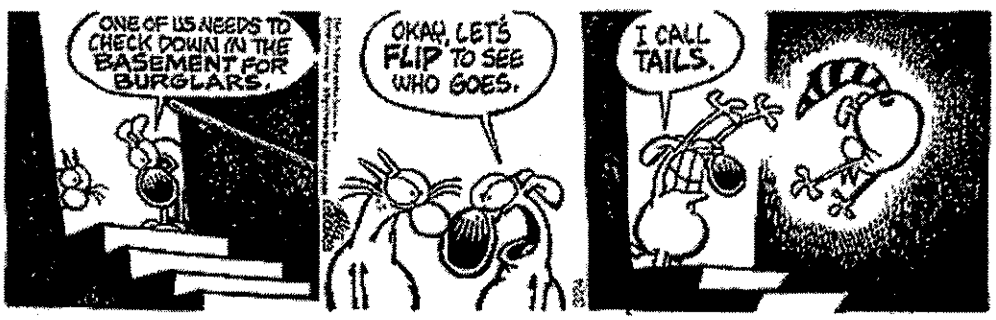
THE STAT FAMILY







MOTHER GOOSE and GRIMM



PROBABILITY RULES (The World)

Definition: The probability [chance] of event A is the proportion [percentage] of the time A is expected to happen when the random process is repeated over and over again.

Opposite Event Rule: The probability that event A happens is equal to one minus the probability that A doesn't happen.

Multiplication Rule: The probability that events A and B both happen is equal to the probability that A happens times the probability that B happens given that event A has occurred.

Definition: Two events are mutually exclusive when the occurrence of one prevents the occurrence of the other.

Addition Rule: The probability that event A or event B happens is equal to the probability that A happens plus the probability that B happens minus the probability that both happen. If events A and B are mutually exclusive, then the probability that event A or B happens is simply the sum of the probabilities.

Definition: Two events are independent if when one happens, the probability that the other happens is unchanged.

Fundamental Counting Principle: If event A can occur in m ways and after A occurs event B can occur in n ways, then the number of ways both events A and B can occur is m x n .

The number of ways k objects can be selected from n objects without regard to order is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Repeated Trials: Suppose we have n independent trials, and the probability that event E occurs in any given trial is p. Then the probability that E will occur exactly k times is

$$\frac{n!}{k!(n-k)!}\, p^k (1-p)^{n-k}$$