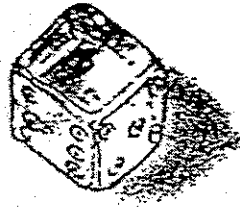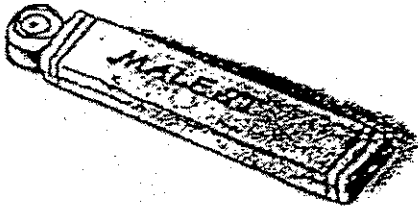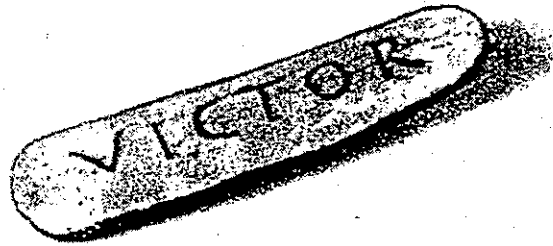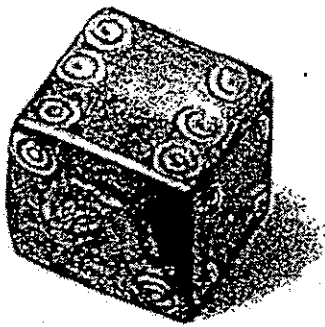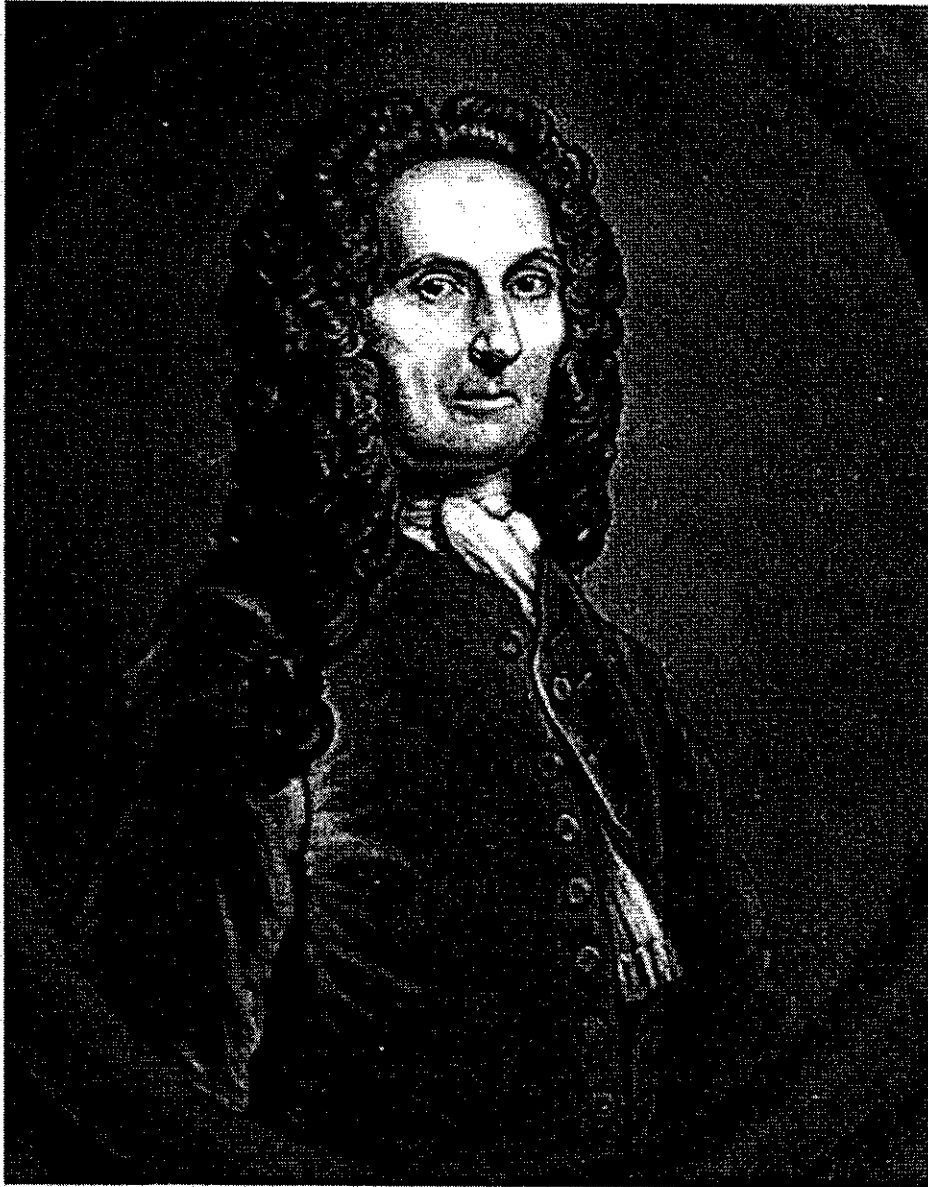**Ancient Greek and Roman Dice:**
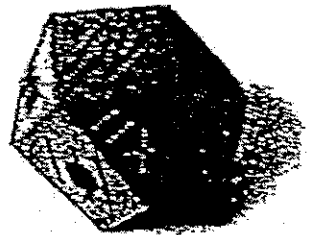
Fermat, France (1601 – 1665)

Pascale, France (1623 – 1662)

Abraham de Moivre (1667 – 1754)

Probability: The Science of Random Events – The Mathematical Analysis of Chance Processes

History of Probability:

Ancient Dice

Jerome Cardan (1501-1576), *The Book on Games of Chance*

Chevalier de Mere, professional gambler (1654)

Blaise Pascal (1623-1662), Pierre Fermat (1601-1665)

Jacob Bernoulli, coin tossing

Abraham De Moivre, *Doctrine of Chances*

Pierre Simon Laplace, *Analytic Theory of Probability*, (1812)
"The most important questions of life are, for the most part, really only problems of probability."

Andrei Kolmogorov, axiomatic approach to probability, (1933)

Chapter 13: What Are the Chances?

Probability was developed to solve gambling problems.

A chance can be:

- a percentage

  the chance a coin lands heads is 50%
- a fraction

  the chance a die lands "6" is 1/6
- a decimal

  the chance a die lands on an odd number is .5

Facts:

- Chances are between 0% and 100% (or 0 and 1 for fractions)

  NEVER NEGATIVE!

  NEVER MORE THAN 100% (or 1 for fractions)

- The chance of something happening is 100% minus the chance of it not happening

  e.g. if the chance you win is 45%, then the chance you don't win is 100% - 45% = 55%

  e.g. if the chance of getting no prizes is .25, then the chance of getting at least one prize is 1 - .25 = .75.

Examples.

1. Roll a die. Your chance of getting either a "1" or a "6" is

   _____

2. A box contains 2 red marbles and 3 yellow ones. You
   reach in and choose a marble at random (without
   looking).

   ● ● ○ ○ ○

   red   red   yellow yellow yellow
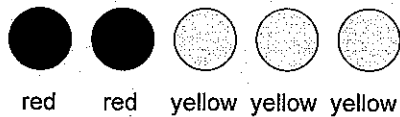
a) The chance the marble is red is _____

b) The chance the marble is yellow is _____

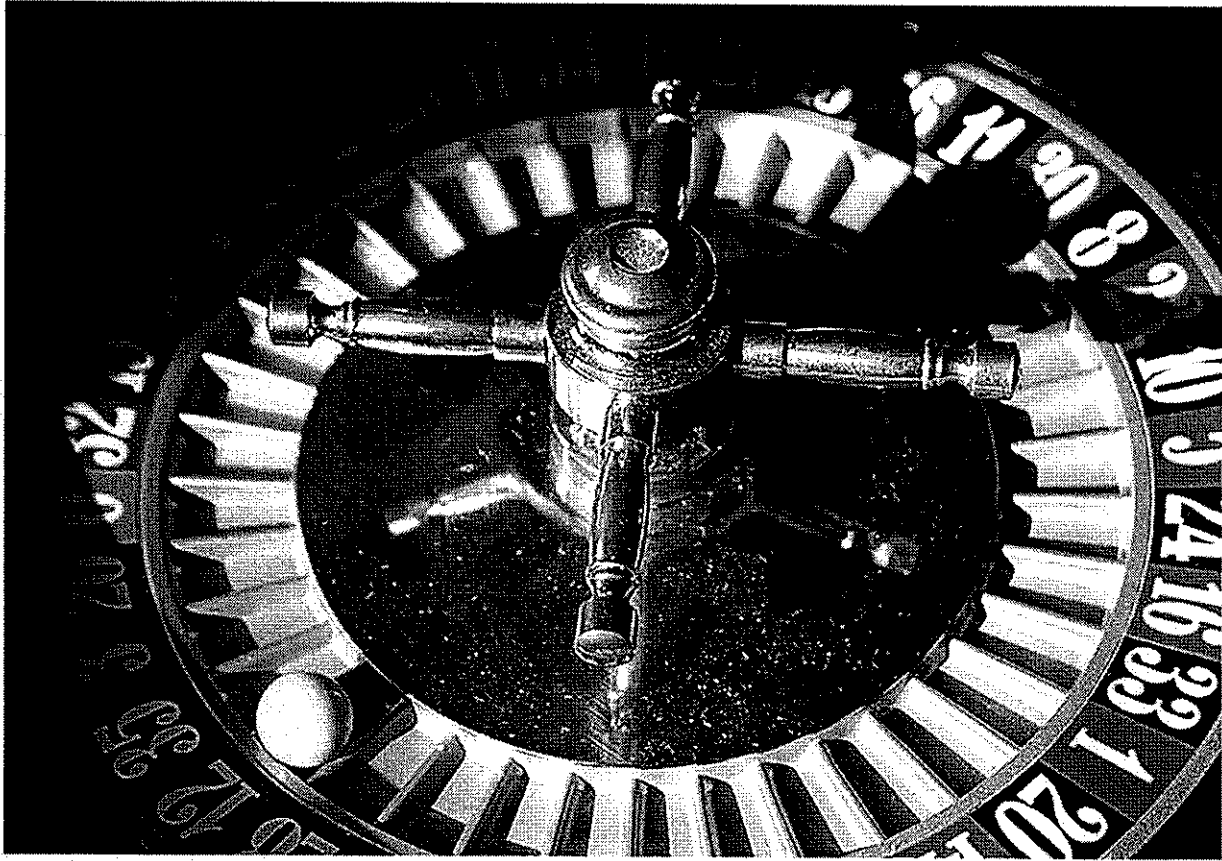American roulette layout and wheel, with single and double zero

3. In **roulette**, we have 38 pockets:

18 red pockets }
18 black pockets } 1 through 36

2 green pockets   0 and 00

If you bet on a number, you win if that number shows up; otherwise you lose. If you bet on red, you win if a red number shows up; otherwise you lose. Etc.

a) If you bet on the number "7", your chance of winning is

_____

b) If you bet on red, your chance of winning is _____

c) If you bet on green, your chance of winning is

_____

d) What is the chance of getting a "7" or a black number? (Note: "7" is red).

4. I have a box containing 1 red die, 2 green dice and a white die. I reach in and choose a die at random, and I roll it.

a) What is the chance I get a green die?

b) What is the chance I **don't** get the green die?

c) What is the chance I get the number "6"?

d) What is the chance I **don't** get the number "6"?

e) What is the chance I get a green die and the number "6"?

f) What is the chance I get a green die or the number "6" (or both)?

5. I toss a coin 2 times.

a) What is the chance it lands "H" both times? Write this as HH.

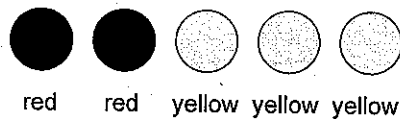b) What is the chance it lands HT?

c) What is the chance it lands TH?

d) What is the chance it lands TT?

e) What is the chance we get at least one H?

Rule: The chance something happens *at least once* = _____

6. A box contains 2 red marbles and 3 yellow ones:

red   red   yellow yellow yellow

Case A: You reach in and choose two marbles **with** replacement.

a) The chance the first marble is red is _____

b) The chance the first marble is yellow is _____

c) The chance the *second* marble is red is _____

d) If we see that the first marble is red, the chance the second marble is red is _____

Case B: You reach in and choose two marbles **without** replacement.

a) The chance the first marble is red is _____

b) The chance the first marble is yellow is _____

c) The chance the *second* marble is red is _____

d) If we see that the first marble is red, the chance the second marble is red is _____

1

1. Draw two cards without replacement from a standard deck. What is the probability that you will draw a *heart* on the first draw? What is the probability that you will draw a *heart* on the second draw? What is the probability that they will both be hearts?

2. Draw twice without replacement from the box [R , W , B]. What is the probability that you will draw R on the first draw? What is the probability that you will draw R on the second draw?

3. There are 390 student in this class and each of you will draw twice without replacement from the box [R , W , B].

   i) About how many of you will get R on the first draw?

   ii) If one-third of the students (130) get R on the first draw, how many of these students will get W on the second draw?

   iii) What is the probability of drawing R on the first draw and W on the second draw?

Multiplication Rule: The probability that events A and B both happen is equal to the probability that A happens times the probability that B happens given that event A has occurred.

PROBABILITY RULES (The World)

Definition: The probability [chance] of event A is the proportion [percentage] of the time A is expected to happen when the random process is repeated over and over again.

Opposite Event Rule: The probability that event A happens is equal to one minus the probability that A doesn't happen.

Multiplication Rule: The probability that events A and B both happen is equal to the probability that A happens times the probability that B happens given that event A has occurred.

Definition: Two events are mutually exclusive when the occurrence of one prevents the occurrence of the other.

Addition Rule: The probability that event A or event B happens is equal to the probability that A happens plus the probability that B happens minus the probability that both happen. If events A and B are mutually exclusive, then the probability that event A or B happens is simply the sum of the probabilities.

Definition: Two events are independent if when one happens, the probability that the other happens is unchanged.

Fundamental Counting Principle: If event A can occur in m ways and after A occurs event B can occur in n ways, then the number of ways both events A and B can occur is m x n .

The number of ways k objects can be selected from n objects without regard to order is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Repeated Trials: Suppose we have n independent trials, and the probability that event E occurs in an given trial is p. Then the probability that E will occur exactly k times is

$$\frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

7. A ticket will be drawn at random from Box A or Box B.

Box A: ■ ■ ☐ ☐ ☐

Box B: ■ ■ ■ ■ ☐ ☐ ☐ ☐ ☐ ☐

You win $1 if the ticket is black. Which box is better for you?

8. Two tickets will be drawn at random *without replacement* from Box A or Box B.

Box A: ■ ■ ☐ ☐ ☐

Box B: ■ ■ ■ ■ ☐ ☐ ☐ ☐ ☐ ☐

You win $1 if both tickets are black. Which box is better for you?

9. A girl reaches in to the following box with her left hand and draws out a marble at random, then reaches in with her right hand and draws out a second marble at random (without replacing the first one).

red   blue   green

a) What is the chance she gets the red marble in her left hand?

b) What is the chance she gets the green marble in her right hand?

c) What's the chance she gets the red in her left hand and the green in her right hand?

10. An elementary school class has 12 boys and 13 girls. Two children are chosen at random to go to lunch with the teacher.

a)   What is the chance that both of the children are girls?

b)   What is the chance that both of the children are boys?

c)   What is the chance that the first is a girl and the second is a boy?

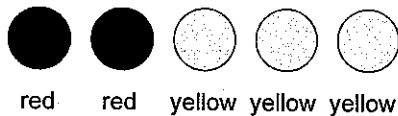d)   What is the chance that the first is a boy and the second is a girl?

10. (continued)

e) What is the chance that one of the children is a girl and the other is a boy?

f) What is the chance the children have the same gender?

g) What is the chance that at least one of the children is a girl?

11. A box contains 2 red marbles and 3 yellow ones.

red   red   yellow yellow yellow

Case A: You reach in and choose two marbles **without** replacement.

a) The chance the first marble is red is _____

b) The chance the second marble is red is _____

c) The chance the first marble is red and the second marble is yellow is:

d) The chance the first marble is yellow and the second marble is red is:

e) The chance one marble is yellow and the other marble is red is:

12. A box contains 2 red marbles and 3 yellow ones:
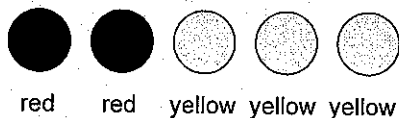
red   red   yellow yellow yellow

Case B: You reach in and choose two marbles **with** replacement.

a) The chance the first marble is red is _____

b) The chance the second marble is red is _____

c) The chance the first marble is red and the second marble is yellow is:

d) The chance the first marble is yellow and the second marble is red is:

e) The chance one marble is yellow and the other marble is red is:

Independence

Two things are independent if the chances for the second are the same, no matter how the first turns out.

Examples:

- Coin tosses are independent of each other
- Rolls of a die are independent of each other
- Draws from a box are independent if we are drawing with replacement
- Draws from a box are NOT independent if we are drawing with replacement
- Cards in a poker hand are not independent

e.g. Toss 2 coins. Are the following things independent?
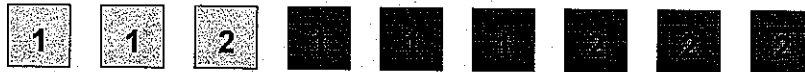
A: getting a H on the first toss

B: getting two H's

13. a) One draw will be made from the following box:
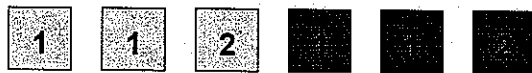
Are color and number independent?
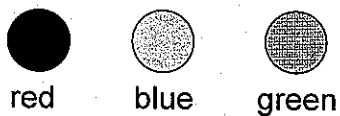
b) As above, but for the box:

c) As above, but for the box:

14. A coin is tossed 5 times. What is the chance of getting HTTHT? What about HHHHH?

15. Roll a die and toss a coin. What is the chance that the die lands 6 and the coin lands T?

16. A girl draws marbles with replacement from the following box:

red   blue   green

a) What is the chance she gets the red marble first?

b) What is the chance she gets the green marble second?

c) What's the chance she gets the red marble first and the green marble second?

d) If she draws 6 times, what is the chance all six draws are red?

e) If she draws 6 times, what is the chance that at least one of the six draws is red?