Math 5710 **Name**______________________

Test 1, Fall 2013

Note: You may leave your answers in uncalculated form.

1. a) From a panel of prospective jurors, 12 are selected at random. If there are 200 men and 250 women on the panel, what is the probability that 10 or more of the jurors are women? (5 points)

10 or 11 or 12

$$\frac{\binom{250}{10}\binom{200}{2} + \binom{250}{11}\binom{200}{1} + \binom{250}{12}\binom{200}{0}}{\binom{450}{12}}$$

b) How many ways are there to arrange a standard deck of 52 cards so that all four aces are next to one another? (5 points)

$(48!)(4!)(49)$

2. Events A, B, C, and D are independent and have respective probabilities $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, and $\frac{4}{5}$. (5 points)

a) What is the probability that exactly three of the events occur? (10 points)

$(A \cap B \cap C \cap D^c) \cup (A \cap B \cap C^c \cap D) \cup (A \cap B^c \cap C \cap D) \cup (A^c \cap B \cap C \cap D)$

$\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{5} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}$

b) What is the probability that at least three of the events occur? (5 points)

3 or 4

answer in a) $+ \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}$

3. Two dice are tossed n times in succession. Compute the probability that "the sum is equal to seven" appears at least once. (10 points)

Probability of "seven" $= \frac{1}{6}$

The opposite event of "at least once seven" is "no sevens"

$$1 - \left(\frac{5}{6}\right)^n$$

4. Suppose that a hand of 7 cards is dealt (without replacement) from a well-shuffled standard 52-card deck. The order in which you hold your cards does not matter. Find the probability of getting exactly three pair: *2 cards from each of three different face values, and 1 more card of still a different face value.*
An example of such a hand is { A, A, 3, 3, 4, 4, J }. (10 points)

$$\frac{\binom{13}{4}\binom{4}{3}\binom{4}{2}\binom{4}{2}\binom{4}{2}\binom{4}{1}}{\binom{52}{7}}$$

5. A box contains 10 balls, 3 are marked WIN and 7 are marked LOSE. You and another player take turns selecting a ball from the box, one at a time. The first person to select a WIN ball is the winner. If you draw second, find the probability that you will win if the sampling is done with replacement. (10 points)

$\boxed{3 \text{ WIN}, \quad 7 \text{ LOSE}}$

How can you win?

LW, LLLW, LLLLLW, ...

$$\frac{7}{10}\cdot\frac{3}{10} + \left(\frac{7}{10}\right)^3\cdot\frac{3}{10} + \left(\frac{7}{10}\right)^5\cdot\frac{3}{10} + \dots$$

Geometric series with $a = \frac{21}{100}$, $r = \frac{49}{100}$

$$\frac{a}{1-r} = \text{sum} = \frac{\frac{21}{100}}{1 - \frac{49}{100}} = \boxed{\frac{21}{51}}$$

6. A bin of 100 electrical components is known to contain 5 that are defective. Suppose the components are to be tested, one by one, randomly, until the defectives are discovered. Find the probability of identifying the last defective on the 10^{th} draw.
(10 points)

$\boxed{95\ Gs,\ 5\ Ds} \rightarrow$ Draw & test without replacement.

4 Ds in 9 draws & then another D

$$\frac{\binom{95}{5}\binom{5}{4}}{\binom{100}{9}} \cdot \frac{1}{91}$$

7. A box contains 4 red tickets, 2 white tickets, and 6 blue tickets. A ticket is drawn, and then it and 8 tickets of the same color are placed back in the box. Finally, a second ticket is drawn.

$\boxed{4R,\ 2W,\ 6B}$

a) Find the probability that the second ticket is blue. (10 points)

B_1 : R on 1st , B_2 : W on 1st , B_3 : B on 1st.

A : B on 2nd

$$P(A) = P(A|B_1)\cdot P(B_1) + P(A|B_2)\cdot P(B_2) + P(A|B_3)\cdot P(B_3)$$

$$= \frac{6}{20}\cdot\frac{4}{12} + \frac{6}{20}\cdot\frac{2}{12} + \frac{14}{20}\cdot\frac{6}{12} \qquad = \frac{1}{2}$$

b) If the second ticket drawn is blue, what is the probability that the color of the first ticket drawn was blue? (5 points)

$$P(B_3|A) = \frac{P(B_3 \cap A)}{P(A)} = \frac{P(B_3)\cdot P(A|B_3)}{P(A)}$$

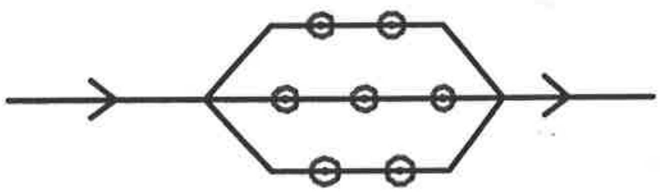
$$\frac{\frac{14}{20}\cdot\frac{6}{12}}{\frac{6}{20}\cdot\frac{4}{12} + \frac{6}{20}\cdot\frac{2}{12} + \frac{14}{20}\cdot\frac{6}{12}} \qquad \frac{7}{10}$$

8. Suppose a test has 25 multiple choice questions: a, b, c, d, e with exactly one of them correct. Being totally unprepared, you decide to guess on every question. Find the probability of getting 23 or more questions correct. (10 points)

23 or 24 or 25

$$\binom{25}{23}\cdot\left(\frac{1}{5}\right)^{23}\cdot\left(\frac{4}{5}\right)^{2}+\binom{25}{24}\left(\frac{1}{5}\right)^{24}\left(\frac{4}{5}\right)+\left(\frac{1}{5}\right)^{25}$$

9. Each component of the following system works independently with probability p. Find the probability that the system works. (10 points)



The system fails if all 3 paths are failures. Otherwise, it works

top fails : $1-p^2$

middle fails: $1-p^3$

bottom fails: $1-p^2$

All 3 fail : $(1-p^2)(1-p^3)(1-p^2)$

System works: $1-(1-p^2)^2(1-p^3)$