**Mathematics 5710**
**Review for Test 3**

1. What is the *joint cumulative distribution function* for two random variables $X$ and $Y$ ?

2. What is the *joint probability frequency function* for two discrete random variables $X$ and $Y$ ?

3. What is *the joint probability density function* for two continuous random variables $X$ and $Y$ ? How is it related to the joint cdf ?

4. How do you compute the probability of an event that is described in terms of two random variables $X$ and $Y$ ? How do you generalize the theory of random variables to higher dimensions?

5. Given the joint frequency function for $X$ and $Y$ , how do you obtain the marginal frequency functions?

6. Given the joint density function for $X$ and $Y$ , how do you obtain the marginal density functions?

7. Given discrete random variables $X$ and $Y$, how do you find the frequency function of $Y$ given $X$?

8. What is the expected value and variance of a discrete random variable?

9. What is the expected value and variance of a continuous random variable?

10. How do you find the variance of a random variable?

11. Suppose you know the joint density function for the continuous random variables $X$ and $Y$. How do you find the density function for the random variable $g(X, Y)$ ?

12. How do you find the expected value of $g(X, Y)$ ? How do you find the variance of $g(X, Y)$?

13. Define what is meant by the covariance of the random variables $X$ and $Y$ ? How do you compute the covariance? What is the correlation $\rho(X, Y)$ and what does it measure?

14. What is the moment generating function of a random variable? Why is it important?

**Review Problems: Test 3**

1. In the gambling game Chuck-a-Luck, for a $1 bet it is possible to win $1, $2, or $3 with respective probabilities 75/216 ,15/216 , 1/216 . One dollar is lost with probability 125/216. If *X* equals the payoff for this game, find E(*X*) and Var(*X*) . Note that when a bet is won, the $1 that was bet is returned to the bettor in addition to the money won.

2. A biased coin with probability of heads equal to 2/3 is tossed until a head appears or 3 tosses have been made. Let *X* denote the number of tosses. Find the mean and variance of *X*.

3. The joint probability density function of $X$ and $Y$ is given by

$$f(x,y) = \frac{1}{8}(y^2 - x^2)e^{-y} \quad \text{for} \quad -y \le x \le y, \ \ 0 < y < \infty$$

Write the density of $X$ in terms of an integral with the appropriate limits of integration. Write the density of $Y$ in terms of an integral with the appropriate limits of integration. Do not evaluate the integrals.

4. A random variable $X$ has the density function

$$f(x) = \begin{cases} 3e^{-3x} & \text{if } 0 \le x < \infty \\ 0 & \text{otherwise.} \end{cases} \qquad \text{Find } E(e^X) .$$

5. Let $X$ be a continuous random variable with probability density function

$$f(x) = \begin{cases} \dfrac{2}{x^2} & \text{if } 1 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$ and $\text{var}(X)$.

6. Suppose $X$ , $Y$ are continuous random variables with joint density

$$f(x,y) = \begin{cases} \frac{1}{2} x e^{-y} & \text{for } 0 < x < 2 \ , \ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Find $P(X \le 1 \ , \ Y \le 2)$.

b) Find $f_X(x)$ and $f_Y(y)$ . Are $X$ , $Y$ independent?

c) Find $P(X^2 + Y^2 \le 4)$. Write your answer as an integral; do not evaluate.

7. The joint probability density function of $X$ and $Y$ and is given by

$$f(x,y) = e^{-(x+y)} \quad \text{for} \quad 0 \le x < \infty \ , \ 0 \le y < \infty$$

Find $P(X < Y)$ , $f_X(x)$ , and $f_Y(y)$ .

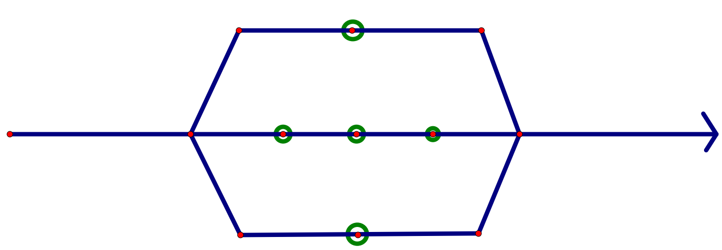
8. Suppose $X$ , $Y$ are independent exponential random variables with parameter $\lambda = 2$.

a) Find $f(x,y)$, the joint density of $X$ and $Y$. .

b) Find $P(X + Y \le 7)$ . Write your answer as an integral; do not evaluate.

9. Suppose $X$ , $Y$ are independent uniform random variables on ( 0 , 2 ) . Suppose $W = X + Y$. Find the density function of $W$.

10. The time until failure of each component in the following system is an exponential random variable with parameter $\lambda = 3$. They operate independently of one another. Let $W$ be the time until failure of the system. Find the density function for $W$.



11. If $X$ and $Y$ are independent random variables with equal variances, find

Cov ( $X + Y, X - Y$ ).

12. Let $X$ be the minimum value when a pair of four-sided dice is rolled. Find the moment generating function for $X$ and use it to find the mean and variance of $X$.

13. Suppose the probability density function of the continuous random variable $X$ is given by

$$f(x) = \begin{cases} 2x , & 0 < x < 1 \\ 0 , & otherwise \end{cases}$$

Find the moment-generating function for X and use it to find the mean and variance of X.